



MAKING A SUCCESS OF MATHS TO 18

OCTOBER 2023

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[Contents]

SECTION 1 – STRATEGY	4
The In-Principle Case for the Study of Mathematics	5
David Thomas, Chief Executive, MESME	
Maths to 18: From Intention to Implementation	8
Prof Andy Noyes, Professor of Education, University of Nottingham	
SECTION 2 – THE MATHS LANDSCAPE	11
Diverging Pathways: Maths Participation in Year 12 in England	12
Carl Cullinane, Director of Research and Policy, The Sutton Trust	
Challenges and Opportunities from the Frontline	18
Ben Mapp, Achievement Director for Mathematics, Harris Academy Chafford Hundred	
Culture and Curriculum	21
Kerry Burnham, Headteacher, Exeter Maths School	
Segmenting the Maths Cohort	24
Gareth Stevens, Chief Executive, Inspiration Trust	
SECTION 3 – DESIGN PRINCIPLES	27
Empowering Girls and Young Women in Mathematics	28
Sherry Coutu CBE, Chair, Maths4Girls	
Increasing A-Level Maths Takeup Among High-Attaining 16 Year Olds	31
Simon Coyle, Head of Philanthropy, XTX Markets	
The Role of Core Maths	34
Prof Paul Glaister CBE, Professor of Mathematics and Mathematics Education, University of Reading	
Maths to 18: The Role of Further Education	37
Sarah Waite, Founder and CEO, Get Further	
SECTION 4 – WORKFORCE	41
The Importance of Providing Routes to Mathematical Sciences Degrees	42
Prof Catherine Hobbs, IMA Honorary Education Secretary, University of Bristol	
Addressing the Teacher Challenge	45
Russell Hobby, Chief Executive, Teach First	
Teacher Professional Learning: An Evidence-Led Approach to Improving Outcomes	48
Marie Hamer, Executive Director, Ambition Institute, and Emma Lark, Associate Director, Ambition Institute	

T	S	Y	Z	E	S	D	C	O	R	Y	L
G	J	T	U	L	P	A	E	X	F	S	K
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Strategy



David Thomas

Chief Executive, MESME

THE IN-PRINCIPLE CASE FOR THE STUDY OF MATHEMATICS

We study subjects because they teach us the knowledge of that subject, and the methods of establishing further knowledge. The case for studying maths is for young people to build their mathematical intuition, improve their fluency and enable them to apply it, so that they see any problem in the world as a model they can manipulate and ultimately solve. A curriculum of maths to 18 ought to be focused on mathematical intuition. This means being able to conduct the process of mathematics - to abstract, to play with variables, to draw conclusions and interpret them - and to do so intuitively.

Many students learn maths because they will go on to become mathematicians. They will lead the next wave of discovery that expands the frontiers of mathematics. But even more students will not. Why, then, should they study maths to 18?

Some argue that it would be in order to teach practical workplace skills. But the range of workplaces that students will go on to is too broad. Nurses, builders and lawyers all need maths, but their maths will take very different forms.

Some argue that it should be about financial literacy. But this misunderstands the financial challenges faced by younger generations as an inability to budget, the maths for which is learned in primary school, rather than, for example, a mismatch between wages and the cost of housing.

And some increasingly argue that, in the age of AI, there is simply no point at all.

This is why I disagree.

Let me start more generally. Why do we study any subject at all? We do so because every subject teaches knowledge about the world, and a new approach to seeing the world. Science, in its broadest sense, teaches both scientific facts (things scientists know about the world) and the scientific method (how these facts have been established and how to advance the discipline by establishing further ones).

The same is true of maths. It teaches knowledge of maths and of how to do maths. Ultimately, the purpose of studying mathematics is for young people to acquire both these types of knowledge; to make them fluent in the facts of maths and how to apply them, so that they can see any problem in the world as a model they can manipulate and solve.

And a curriculum of maths to 18 could transform how future generations see the world by imbuing them with mathematical intuition.

Maths is all about abstraction. It is about taking the concrete world and representing it in a model that is manipulable. This lets you ask questions about things you have not yet observed. It is what primary school children do when they ask "What if there were two fewer?" and what researchers do when they ask "What if there were another dimension?"

Maths teaches you to approach the world as a model. That even the toughest problems can be solved if you model them in a way that helps you to expose what is going on under the surface, and to start to test and refine possible solutions.

But is this still relevant in the age of AI? Computers have, of course, been able to do maths for a long time. Plenty of free tools have long been capable of doing every schoolchild's maths homework. But now that computers can do natural language processing, they can not only compute the mathematics, they can do the abstraction and interpretation. If that is the case, why learn mathematics?

There is a difference between being able to ask something of a machine and being able to work it out yourself. We learn subjects to gain knowledge about the world, and approaches to seeing the world. Outsourcing to a computer does not change how you see the world.

Our goal should be to change how future generations see the world by building their mathematical intuition. This is the ability to conduct the process of mathematics - to abstract, to play with variables, to draw conclusions and interpret them - and to do so intuitively.

While it might be a lovely ideal to say that we could, in a world of omniscient AI, simply ask it a question and follow its answer, we know that humans do not behave in this way. We follow our intuitions. We see examples of this all the time, where our intuitive biases control our behaviour rather than logical argument or fact. If you want to get young people to save more money, do not teach them budgeting, give them an intuition for exponential growth!

This intuition operates at times when you cannot frame a question and go out to the AI; when you are at the negotiating table, in a crisis, or just all those small situations where it is easier to go with your gut, rather than to go and find the definitively correct answer.

A curriculum for maths to 18, then, should be a curriculum to build mathematical intuition. The A-Level does this within pure maths, statistics and mechanics. But those students who do not take A-Level Maths need an alternative that builds on the academic foundations from GCSE and takes them further. This would be, essentially, a curriculum in mathematical modelling.

Under this approach, students would learn about how to construct a mathematical model. They would learn how to consider which variables to include and which to exclude or hold fixed. They would learn how to make and record assumptions in a deliberate way so that they can come back and change these later.

They would learn how to decide which mathematics to use within their model. They would learn how to decide whether variables change linearly or logarithmically, and how these might interact. They would learn how to approximate answers, perhaps by estimation or perhaps by sketching a graphical relationship.

They would also learn how to interpret and use the output of their modelling. They would learn how to put bounds on an answer and how to give a degree of confidence in a finding. And they would learn to hold their model to account based on whether it gives a useful answer, not just on whether they have done the maths correctly.

Much of the content knowledge that students would need to do this will have already been learned by the end of GCSE. We would need to build on their understanding of algebraic relationships, and likely add some content on logarithmic relationships. But the biggest area of new mathematical content would be data and statistics.

Students need both to know how to use a probability distribution and to develop an intuition for things like confidence intervals. Without this, people would struggle to model so many of the meaningful problems involving large data sets that we face in the world today.

An entitlement of studying maths to 18 could be transformative for the UK, but only if we get it right. The opportunity here lies in developing mathematical intuition, where young people have become fluent enough in maths and its applications to be able to see any problem as a model they can manipulate.

Generations of people who think this way will be better able to solve the world's greatest problems than any generation that has come before them.



Prof Andy Noyes

Professor of Education, University of Nottingham

MATHS TO 18: FROM INTENTION TO IMPLEMENTATION

Maths to 18 as a policy concept has been around for some time. It is almost 20 years since the original Smith Report set out an ambitious agenda for change around different pathways for maths from age 14 onwards. Since then, changes to wider education systems and curricula, including things like the dropping of AS-Levels, have meant that such a goal has become harder to achieve. The policy and stakeholder buy-in for maths to 18 is there, but the challenge remains implementation. Few challenges are as operationally difficult as maths to 18 would be. Compulsion would be perhaps the most, or maybe the only, way to do so, but it is replete with perverse incentives.

It is almost 20 years since Prof Sir Adrian Smith's report on 14-18 maths education, *Making Mathematics Count*.¹ Commissioned by the Qualifications and Curriculum Authority, the report set out an ambitious agenda for change in upper secondary maths education, including a vision for 'mathematics pathways':

We wish to see a highly flexible set of interlinking pathways that provide motivation, challenge and worthwhile attainment across the whole spectrum of abilities and motivations but avoid the danger of returning to the O-level / CSE "sheep and goats" divide.

Although the pathways metaphor has become embedded in research and policy discourse, the report's ambitions remain largely unfulfilled.

Over the ensuing years, a major programme of curriculum and qualifications design took place. It included a double GCSE and the expansion of the then AS-Level 'Use of Mathematics' (UoM) to a full A-Level. In addition, a GCSE UoM for low attainers in further education (FE) colleges was piloted, acknowledging that their learning and engagement needs were different. Those pathways were designed to address the same problems under discussion today, and the evaluation of the pathways programme highlighted some of the challenges of systemic change.²

Enthusiasts of UoM welcomed the full A-Level, defending its unique appeal to a group of learners, many of whom found a new love of usefully applied mathematics. Similarly, FE students found the modular approach of GCSE UoM to be relevant, reinvigorating and engaging, and it even included some financial maths. Sadly, the promise of those qualification developments was unrealised. Before long, an assault on UoMs qualification precipitated their demise.³ While the qualifications were not perfect, they were highly valued by those committed to their principles and pedagogic approaches.

Smith's commitment to maths to 18 was reinforced by the Secretary of State shortly after the change of government in 2011, in part influenced by the Nuffield Foundation's comparative work in this area, and by Alison Wolf's report on vocational education.⁴ Speaking to the Royal Society, Michael Gove set out "a new goal for the education system so that within a decade the vast majority of students are studying maths right through to the age of 18."⁵

Drawing on advice from the Royal Society's Advisory Committee on Mathematics Education (ACME), Core Maths was introduced in 2014.⁶ It was soon evident, however, that the new qualification's promise of fulfilling the shortfall of circa 250,000 students who achieved a GCSE grade C / level 4 and then stopped studying maths would not be met.

It is instructive to revisit the 11 recommendations of ACME's "Planning for Success", as they remain highly relevant. Indeed, it could be argued that some of the limitations of subsequent policy moves result from the partial application of these recommendations.

A few years after Core Maths was introduced, AS-Level and A2-Level qualifications were decoupled, together with a general move from starting four A-Levels to three. This resulted in the loss of a large number of students who were only completing AS-Level Maths. It is a good example of the contradictions and negative unintended consequences of uncoordinated policy moves.

1. Smith, A. (2004) "Making Mathematics Count: The report of Professor Adrian Smith's Inquiry into post-14 Mathematics Education". The Stationery Office. <https://dera.ioe.ac.uk/id/eprint/4873>
2. Noyes, A., et al. (2010) "Evaluating Mathematics Pathways: Final Report". Department for Education. https://assets.publishing.service.gov.uk/government/uploads/system/uploads/attachment_data/file/182076/DFE-RR143.pdf
3. Truss, L. (2009) "Liz Truss: The Misuse of Maths". Conservative Home. <https://conservativehome.blogs.com/centreright/2009/07/elizabeth-truss-the-misuse-of-maths.html>
4. Hodgen, J., et al. (2010) "Is the UK an outlier? An international comparison of upper secondary mathematics education". Nuffield Foundation. <https://www.nuffieldfoundation.org/about/publications/is-the-uk-an-outlier-an-international-comparison-of-upper-secondary-mathematics-education>
5. Gove, M. (2011) "Michael Gove speaks to the Royal Society on maths and science". Department for Education. <https://www.gov.uk/government/speeches/michael-gove-speaks-to-the-royal-society-on-maths-and-science>
6. For example, ACME. (2012) "Post 16 mathematics: a strategy for improving provision and participation". <https://royalsociety.org/~media/policy/Publications/2012/post-16-mathematics-a-strategy-for-improving-provision-and-participation-12-2012.pdf>

Ironically, the loss of AS-Level Maths coincided with the unusual step of HM Treasury commissioning another report on post-16 maths education, the second by Prof Sir Adrian Smith.⁷ Published in 2017, the motivation at this point was much more explicitly an economic one. Smith reinforced Gove's earlier maths-to-18 aspiration, with a caveat:

In relation to the issue of most or all students continuing mathematics until 18, my clear conclusion is that we do not yet have the appropriate range of pathways available or the capacity to deliver the required volume and range of teaching. However, I would hope that if we were able to move forward over the next few years with many of the recommendations in this report, we might realistically aspire to such a vision within a decade.

Seven years into Gove's decade, the pathways model that 2004's "Making Mathematics Count" had called for was not in place, and major questions about capacity and infrastructure remained unresolved. Nevertheless, the maths to 18 intention was clear, not just in Gove's speech to the Royal Society, but in a major Treasury commissioned report.

Curriculum policy in the late 2010s was dominated, however, by qualifications reform, leaving little space for the kinds of carefully designed, developed and implemented strategy necessary for achieving this maths to 18 goal. Many of these reforms aimed to make the maths qualifications more demanding, not more inclusive, prioritising advanced techniques rather than mathematical literacy (of the kinds the OECD emphasised in PISA assessments). The decade had seen major steps backwards and forwards, but little overall progress.

In other words, the intention of maths to 18 is supported, as it has been for some time, by a wide range of key policymakers and stakeholder groups. It is also likely that the kinds of ideas in play for 20 years are probably about right, such as different pathways and new qualifications for middle attainers. But there remains now, as there has for some time, little by way of long-term, strategic planning that is likely to realise this vision. The Maths to 18 Expert Advisory Group has a challenge ahead of it!

In particular, there appears to be a lack of good implementation science and understanding of complex system change to make the move. There are few step changes in education comparable with getting circa 250,000 students who currently do not want or have to study maths beyond 16 to do so. Some research, drawing on a survey of over 10,000 17-year-olds, concluded that the only way to make maths to 18 happen would be through compulsion.⁸ However, the resulting conundrum is that this would almost certainly exacerbate negative societal attitudes to mathematics, something which was very much a concern of Smith's 2017 report.

It is also important to note what else has happened in schools since 'mathematics pathways'. Namely, two decades of gradual fragmentation of the education system, which has arguably made change more challenging. That shift is not about maths per se, but it is framing context for any maths to 18 plan, and so policy designers and implementation planners will need to be adept at navigating this contorting educational landscape.

The reality is that the solution to post-16 maths will not simply be found in post-16. As our work for XTX Markets found, a genuine commitment to more engaging post-16 maths pathways for all learners requires a fundamental rethink of how learners get to that point, and in particular their experiences from 11-16.⁹ We cannot deal with maths to 18 in isolation.

7. Smith, A. (2017) "Report of Professor Sir Adrian Smith's review of post-16 mathematics". Department for Education. https://assets.publishing.service.gov.uk/government/uploads/system/uploads/attachment_data/file/630488/AS_review_report.pdf

8. Noyes, A., and Adkins, M. (2016) "Studying advanced mathematics in England: findings from a survey of student choices and attitudes". Research in Mathematics Education, 18 (3). <https://nottingham-repository.worktribe.com/output/796951/studying-advanced-mathematics-in-england-findings-from-a-survey-of-student-choices-and-attitudes>

9. Noyes A., et al. (2023) "The mathematics pipeline in England: patterns, interventions and excellence". <https://www.datocms-assets.com/10954/1679582918-xtx-markets-maths-excellence-pathways-230323.pdf>

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The Maths Landscape



Carl Cullinane

Director of Research and Policy, The Sutton Trust

DIVERGING PATHWAYS: MATHS PARTICIPATION IN YEAR 12 IN ENGLAND

For social mobility purposes, it is important that a maths to 18 policy benefits those from more disadvantaged backgrounds. Using recent longitudinal data, we can analyse who currently studies maths among those who have just finished their post-16 education. We observe two distinct groups who study maths at different levels (GCSE and A-Level) at age 16-18. Students who come from lower socio-economic status backgrounds are less likely to study A-Level Maths, and are more likely to be retaking GCSE Maths. Policymakers must ensure that maths to 18 does not widen these participation gaps.

Back in 2013, the Sutton Trust released a report titled 'The Employment Equation'.¹⁰ It welcomed a recent announcement requiring young people to continue to study maths if they had not achieved a pass at GCSE, but urged the government to go further, recommending that all young people should study maths to 18.

A decade later, this issue has shot to the top of the agenda, and one of the paper's authors sits on the Maths to 18 Expert Advisory Group appointed by the Prime Minister to explore implementation of the policy; a neat illustration of the winding road that achieving policy change can take.

The Sutton Trust's social mobility mission has meant a laser focus on ensuring that opportunities for advancement are open to young people from all backgrounds. With regards access to maths education, this mission is no different. Given the clear benefits of maths, both in terms of basic numeracy and higher-level skills, it is vital that people from all backgrounds can access quality maths teaching and fulfil their potential.

Britain has a dual problem: low levels of maths attainment generally, and also clear social inequalities in the pattern of progression to advanced maths qualifications. Any move to maths to 18 should ensure that both issues are tackled.

The new COVID Social Mobility and Opportunities (COSMO) longitudinal study is a survey of over 13,000 young people in England, many of whom have just sat their A-Level exams.¹¹ It highlights some important detail on the characteristics of those currently studying post-16 maths, and helps us to consider how to deliver maths to 18.

16% of the full cohort report studying A-Level Maths or Further Maths in Year 12. This rises to 18% including AS-Levels. About 5% of the full cohort report taking or retaking GCSE Maths. Within the A-Level cohort specifically, the proportion of students taking A-Level Maths or Further Maths is about 28%.

The following tables and analysis explore maths participation in Year 12 in England, looking at student characteristics, household characteristics, school characteristics and prior attainment.

10. Hodgen, J., and Marks, D. (2013) "The Employment Equation". Sutton Trust. https://www.suttontrust.com/wp-content/uploads/2019/12/MATHSREPORT_FINAL-1.pdf

11. Sutton Trust. (2023). "COSMO Longitudinal Study". <https://cosmostudy.uk/>

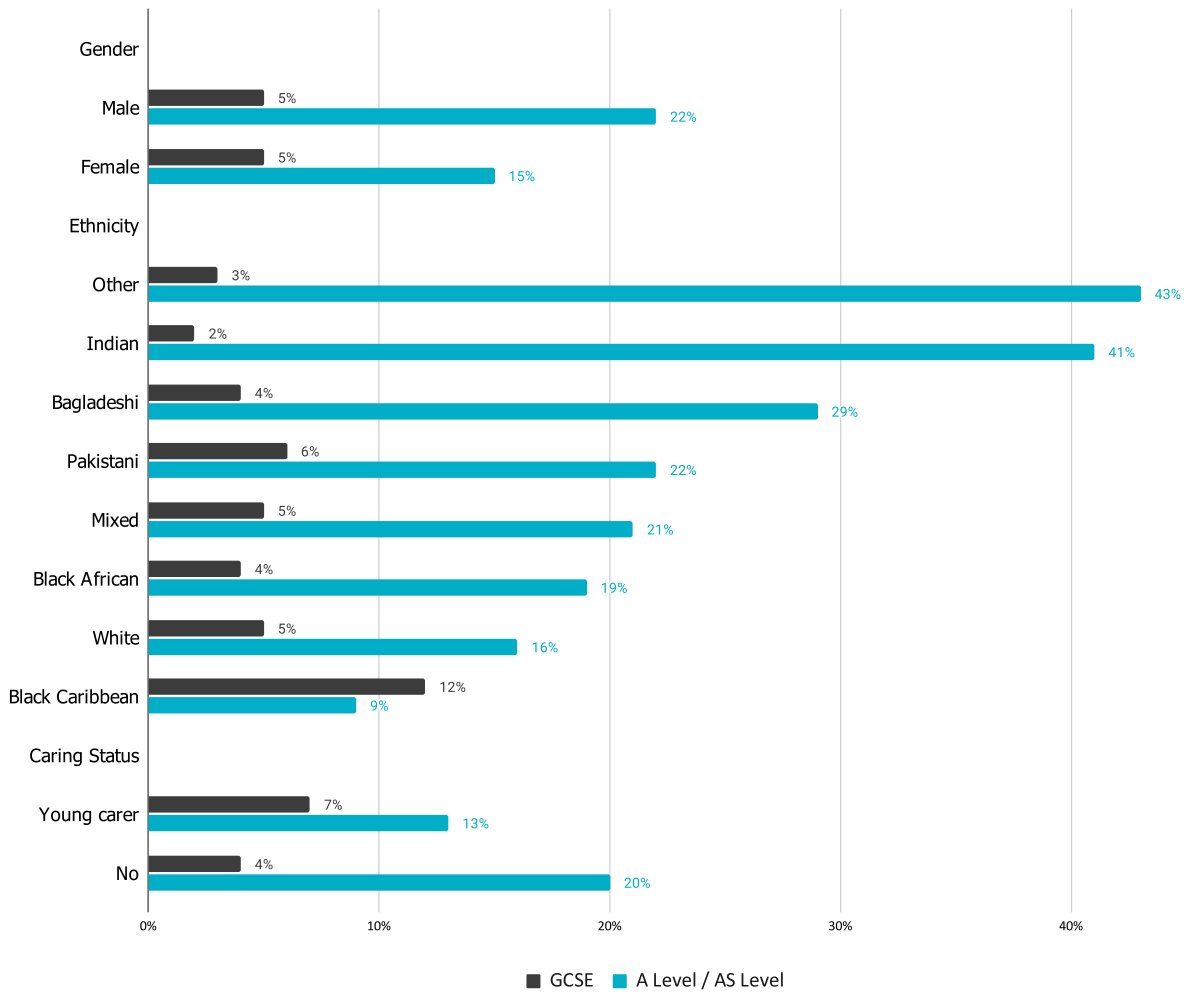


Fig 1: Participation in maths in Year 12, by student characteristics (Source: COSMO longitudinal study)

As reflected in the University of Nottingham’s maths pipeline report, gender plays an important role in post-16 maths participation. 22% of male students report taking AS-Level or A-Level Maths, compared to 15% of female students.

There is also wide variation by ethnicity. Black Caribbean students are least likely (9%) to report taking AS-Level or A-Level Maths, followed by white students (16%). Conversely, 41% of Indian students and 43% of students of ‘Other’ ethnicities (which includes Arab and Chinese students) report taking maths at A-Level. Students who are carers are seven percentage points less likely to report taking A-Level Maths than non-carers.

Black Caribbean and White students are most likely to be taking or retaking GCSE Maths, and students who are carers are almost twice as likely as non-carers. The gender balance is even.

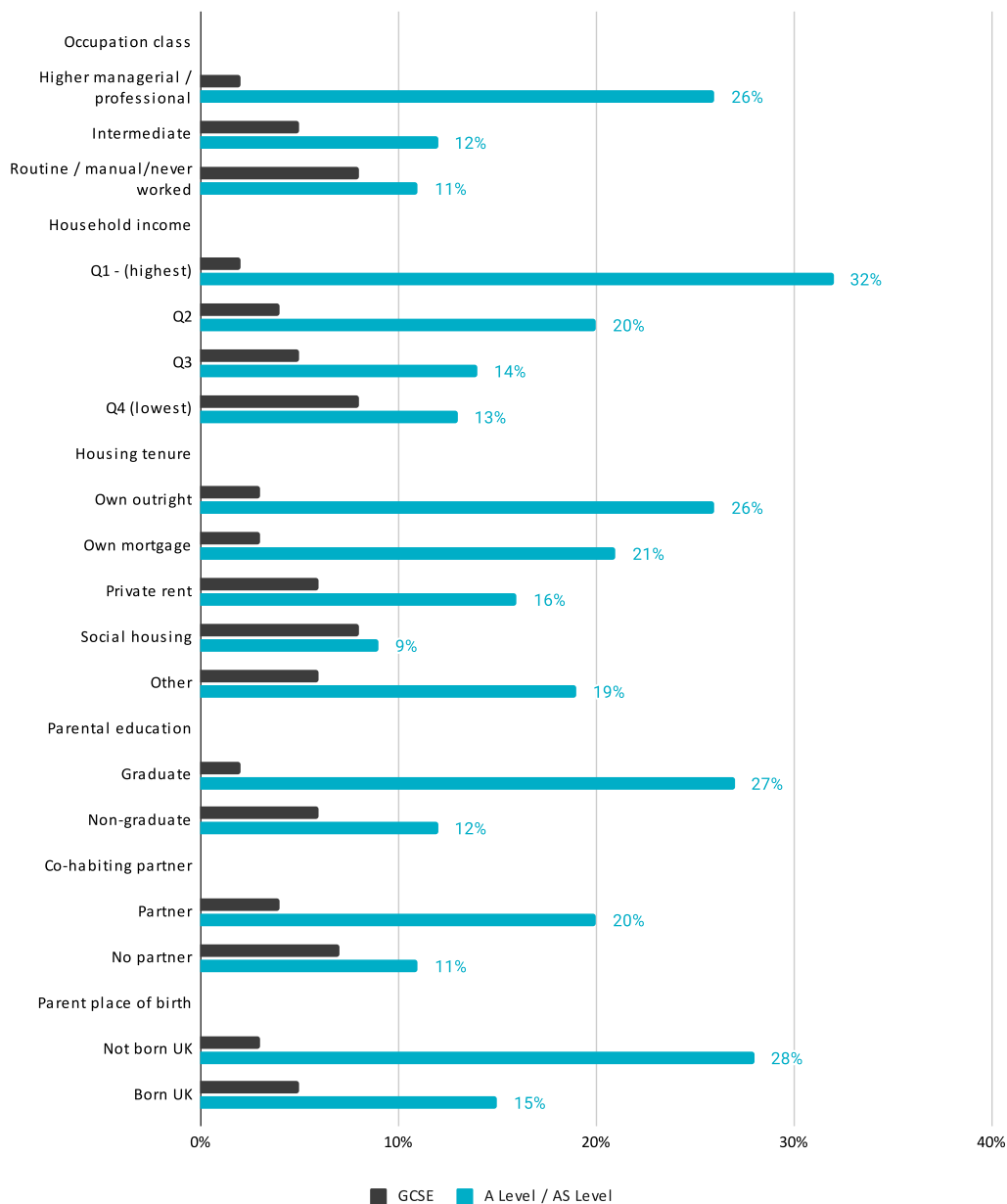


Fig 2: Participation in maths in Year 12, by household characteristics (Source: COSMO longitudinal study)

Participation differs by household factors, especially socio-economic status. Students from managerial and professional class households are most likely to take AS-Level or A-Level Maths (26%). They are more than twice as likely to do so as students from routine or manual class households. There is a gap of 19 percentage points between students from the highest and lowest income quartiles.

Similar patterns are seen by housing tenure, parental education and whether a student's parent lives with a co-habiting partner. Reflecting the patterns seen by ethnicity, students with a parent not born in the UK have a much higher rate of participation than those with a UK-born parent.

Again, patterns in each case are exactly the opposite for taking or retaking GCSEs. For example, students from routine or manual class households are almost four times more likely than students from professional class households to be taking or retaking GCSEs.

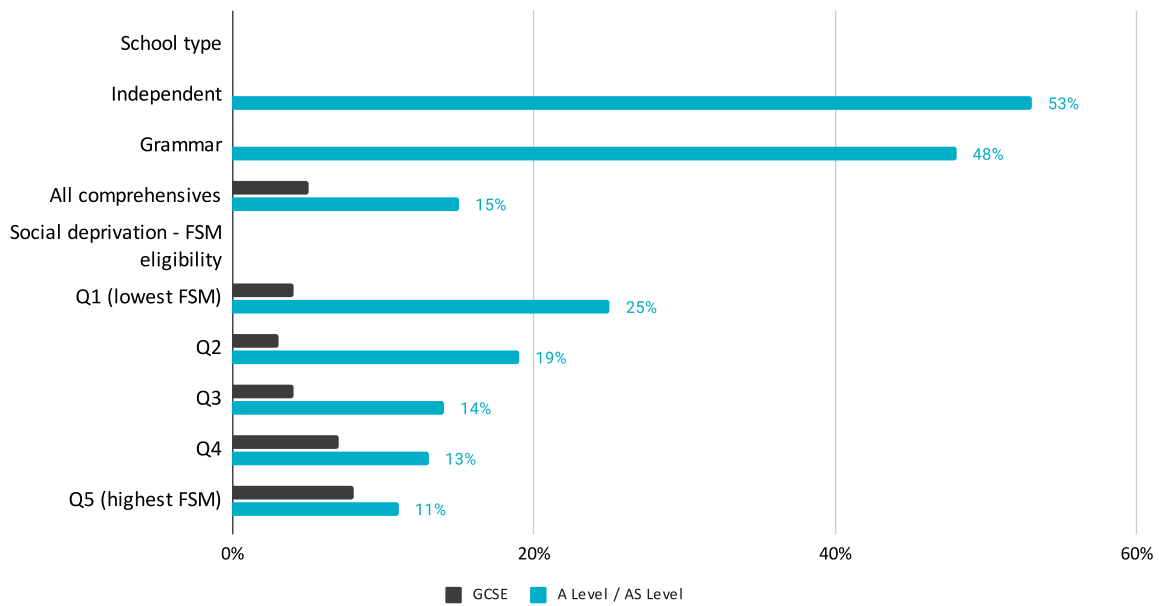


Fig 3: Participation in maths in Year 12, by school characteristics
(Source: COSMO longitudinal study)

Independent schools and grammar schools have extremely high levels of AS-Level and A-Level Maths participation, more than triple that of comprehensive schools. Within comprehensive schools, the least deprived schools (by FSM eligibility) have higher participation levels. In terms of taking or retaking GCSE Maths, the most deprived schools have twice the level of the least deprived, and independent schools and grammar schools have virtually zero GCSE entries in Year 12.

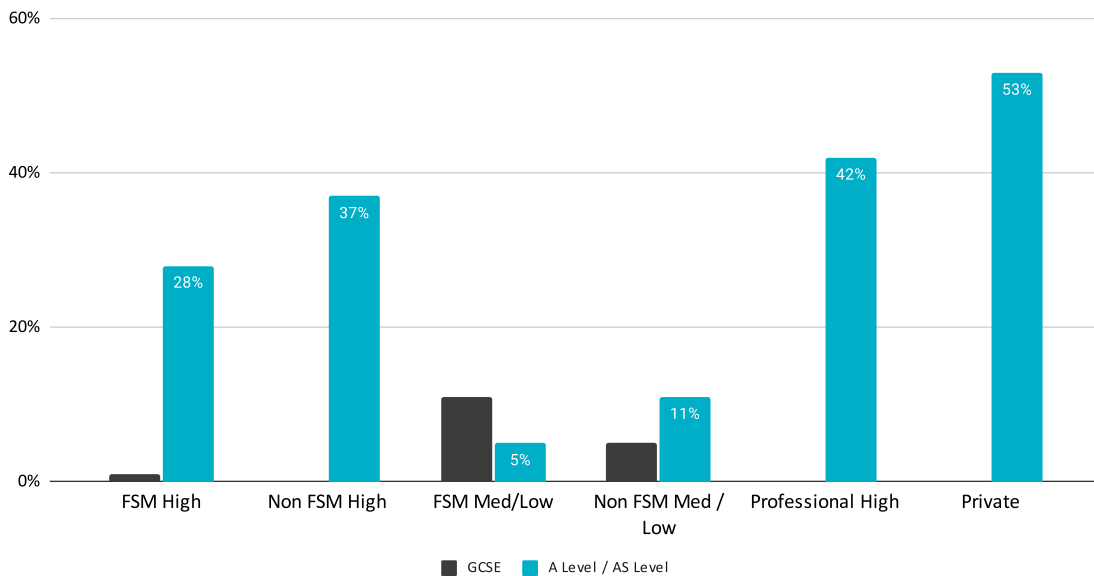


Fig 4: Participation in maths in Year 12, by FSM and Key Stage 2 attainment
(Source: COSMO longitudinal study)

The above statistics do not account for prior attainment in maths, which is a major driver of post-16 pathways. Figure 4 therefore looks at participation in maths in Year 12 by FSM eligibility and by broad bands of attainment at Key Stage 2. FSM-eligible high prior attainers are less likely to take AS-Level or A-Level Maths compared to non-FSM-eligible students of similar ability, and much less likely than those from professional class households or those attending independent schools. As expected, GCSE Maths retakes are more common among low or medium prior attainers. Even so, FSM-eligible students are still twice as likely to retake GCSE maths as non-FSM-eligible students.

The above analyses are descriptive in nature and bivariate only, which obscures the complex interactions of student and family characteristics that impact on participation. A series of logistic regressions were therefore also run to understand which factors had the strongest relationships with participation in A-Level Maths, controlling for other factors. The findings show that:

- Ethnicity was consistently associated with differences in participation levels. In particular, Indian (odds ratio 2.3), Bangladeshi (2.2) and 'Other' ethnicities (2.8) all had more than double the odds of participation of white students. These relationships were largely unchanged when an indicator of high Key Stage 2 attainment was included.
- Girls had half the odds of boys of participation, reducing to 0.4 when prior attainment was included.
- Grammar school students had double the odds of participation compared to the most deprived comprehensive school students. However, this association was no longer significant when attainment was controlled for.
- The strength of the relationship for attending an independent school increased when attainment was controlled for, with an odds ratio of 3.7 compared to the most deprived comprehensive school students
- The level of parental education was also associated with participation levels. Those without a graduate parent had an odds ratio of 0.6 compared to those who did, also robust to the attainment indicator.
- Finally, those with a UK-born parent had an odds ratio of 0.6.

In conclusion, these data show the diverging pathways of young people in post-16 maths. Therefore, while improving the overall level of participation in post-16 maths is likely to be beneficial, it is important to consider the distribution of who is taking which qualifications, as well as what options are offered to those of different ability levels.

The goal should be to offer students at all levels options that stretch them, but also recognise their achievements and facilitate progress to the next steps of their education and career. For social mobility purposes, it is vital that lower prior attainers, who are more likely to be from disadvantaged backgrounds, are properly catered for.



Ben Mapp

Achievement Director for Mathematics,
Harris Academy Chafford Hundred

CHALLENGES AND OPPORTUNITIES FROM THE FRONTLINE

Our school, and many others, provide evidence that children from even the most disadvantaged backgrounds, and that have no parental history of higher education, can achieve in maths and progress to successful careers. For schools to buck the wider trends of maths attainment and progression, we think that there are four lessons we can share: the need to recognise that maths to 18 will encompass young people with a range of different abilities; the importance of supporting the teacher workforce; the need not to neglect primary education; and the power of linking maths to future careers.

As Achievement Director for Mathematics in a non-selective secondary school in Thurrock, Essex, I have had the privilege of seeing students from a wide range of backgrounds go on to start successful careers. Many of these have been in architecture, business, finance, IT, research, and yes, even teaching. These opportunities are, in no small part, due to their attainment in maths.

Such positive outcomes are possible in Thurrock despite pockets of very high deprivation, and lower than average numbers of students going to Oxbridge or Russell Group universities.¹² We have helped many students to overcome significant disadvantage through a relentless drive to know every child and to deliver a rich maths curriculum. We encourage a thirst for knowledge, raising our students' aspirations beyond their lived circumstance.

But there is no denying that it is a challenge to address the wider barriers around maths education, and to generally seek to achieve educational excellence in an area which can seem hostile to it.

As the maths lead at our school, I draw four conclusions as to how we, and I am sure other schools, seek to do it.

First, prioritise the student experience, recognising that they will approach post-16 maths from different perspectives. One of the great things about being a teacher is that you get to work with so many unique young people, all with their own life journey. To encourage these individuals, with enthusiasm and imagination, to build an appreciation for maths is a privilege.

Students clearly possess a spectrum of attitudes to maths. Some do not care much for the subject, some enjoy it but do not see themselves progressing beyond GCSE, and some love maths so much that they are certain they want to study it post-16. Some of those students will want to progress on to A-Level Maths and Further Maths, and then perhaps to undertake a maths degree, or a related one such as economics or engineering.

Along with these differing attitudes to further study, I teach students with a range of prior attainment. Some find almost all aspects of maths a struggle, and others find it accessible, logical and straightforward. In a world where all these young people carry on with some form of maths to 18, these differences would matter, and even magnify.

However, I want to emphasise that young people are not fixed to these groups. With considerate, enthusiastic and expert teaching, students can build confidence and make incredible progress. I have seen students who started their maths careers in our middle sets and ended up succeeding in top set, achieving grade 8 or 9 in GCSE Maths. Many of these students will have made two, three or even four grades of progress above national expectations, making the transformative, life-changing difference that all teachers hope to see their students achieve.

Second, nurture the teacher workforce. The teacher experience is just as varied and rich as the student experience. Some – many – teachers have had no formal mathematical training beyond school level, and some – fewer, sadly – have completed multiple degrees in maths. Many teachers have several years of experience under their belts; others are new to teaching. But all teachers, regardless of their starting point, need to see a progression in their professional development, raising their ability to guide the next generation of mathematicians.

I work with a team of dedicated and diligent teachers, who care deeply about their subject and constantly reflect on how they can improve as teachers and mathematicians. I suspect that we are fortunate in the support from our school, with significant time for development and training. However, given the extreme shortages of staff, I can understand schools requiring teachers to teach the absolute maximum of lessons, giving less time for planning and developing pedagogy.

This is a false economy. One reason that we retain highly qualified and experienced staff is that we provide excellent training opportunities. This is an investment that pays off! Funding and supporting schools to increase continuing professional development in maths will pay dividends more broadly, because it will create better maths teachers who are more likely to stay in the profession.

12. Thurrock Council. (2023) "Thurrock facts and statistics 2023". <https://www.thurrock.gov.uk/thurrock-facts-and-statistics/overview>; and Montacute, M., (2018) "Access to advantage". Sutton Trust. <https://suttontrust.com/wp-content/uploads/2018/12/Access-to-Advantage.pdf>

Furthermore, access to high-quality and practical curriculum resources is vital. Again, we are fortunate to have a large MAT providing excellent, in-depth and refined teaching resources and plans. This will not be the same for all schools. The DfE non-statutory guidance is a good start, but the documentation is monolithic and could be more accessible. With a modest investment, this can become a practical resource for all departments to improve their curriculum. If schools are going to deliver maths to 18, retaining teachers and developing their subject knowledge is essential.

Third, do not forget primary. Outstanding secondary schools can close gaps, break the tyranny of the postcode lottery, and give disadvantaged children a future. Despite this, one of the biggest predictors of progression to A-Level Maths is success at primary level. Within one week of working with our new Year 7 intake, I can identify with a good degree of accuracy the students that are likely go on to A-Level Maths. Sure, there will be many students who find their love for maths in later years, but the reality is that early success matters.

More children will take maths to 18 if primary school teachers are supported with subject knowledge enhancement and curriculum development. Providing funding to place primary school maths leads in a secondary school once a week is one example of a structured programme that could have a significant impact. Primary school teachers need to see regular secondary teaching and vice-versa. If we want to create more young mathematicians who aspire to study post-16 maths, we need to start early and support teachers across the primary-secondary spectrum.

Fourth, the importance of practicality. David Thomas is right in his introductory essay about the in-principle joy and beauty of mathematics as a discipline. But for many teachers and schools, the priority is to engage on a practical basis.

I have seen how attitudes to maths can be dramatically improved by showing how maths makes a difference in real life. Most young people have not yet owned a credit card, had to deal with salary negotiations, completed a mortgage application, or had to plan and run a budget. Better-off students often receive guidance from family members, but where do disadvantaged students get this guidance from? Continued maths education has a role to play.

We should also be clear about the importance of maths careers. My experience is that those who have family-friendly professional networks often have a high motivation to study maths. Such networks might be in engineering, finance, IT or other sectors, helping young people to get a foot in the door. The challenge is to give these opportunities to all.

One option to support this might be a vocational post-16 maths qualification, delivered through modular study and assessable online. The certification would demonstrate that a young person understands the practical maths used in various professions. It would be vocational because it links directly to real world careers. It would be highly regarded because it signals to employers that qualification holders can apply mathematical thinking to their job.

We need to be unapologetically enthusiastic about careers using maths. We should show our students marine shipping insurers who price up the risk of transporting cargo through a warzone. We should show them video game creators who calculate collisions. We should show them Google employees who need to summarise A/B test results. Careers must be at the heart of any post-16 maths programme.

The impact of supporting maths learning cannot be underestimated. However, it must be considered through the lens of students and teachers, from early years to careers. When these elements work in unison we can deliver a transformative and powerful curriculum, for all.



Kerry Burnham

Headteacher, Exeter Maths School

CULTURE AND CURRICULUM

Ensuring a strong pipeline of willing 16-18 maths learners requires action on culture and curriculum alike. The former is hard, but we have time. The latter builds on a series of things we know about suitable curricula, as well as the labour force to deliver it. The network of specialist maths schools, set up to teach 16-18 maths, and with a mission to support access to mathematics for students from under-represented groups, can help. U-Maths, a collaborative new initiative between the specialist maths schools, can also play a role.

Comments one rarely hears:

"I just don't have an English-brain."

"I never did get on with reading in school."

"Students shouldn't be made to read and write post-16. It's cruel. Let them do what they enjoy."

And yet, substitute maths in place of English, reading or writing, and these are socially acceptable, common declarations. They stem from a sense of fear about mathematics; from the belief that not everyone is capable of learning mathematics, to a perception that mathematical skills, beyond basic numeracy, are not necessary or relevant for everyone.

Such cultural attitudes are deeply entrenched in Britain. If not addressed, compelling all students to study maths to 18 will be strongly opposed by students, parents and educators. This is not a post-16 issue. Addressing attitudes to maths should start with the youngest children, their families and their teachers.

Change must start in pre-school, supporting children and their parents to develop numeracy as the foundation for mathematical study. It must continue through primary school. Teaching for mastery is beginning to have an impact, but it is variable. Investment should continue so that all classrooms have a teacher who believes in and supports all students to make progress.¹³ In secondary schools, the curriculum must build on the foundations laid at primary by teaching for mastery, and maths teachers must reassess their beliefs about their students' abilities.

The Key Stage 4 curriculum must be appropriate and relevant. It may benefit from review. The current GCSE Maths qualifications are testing two very different things: basic numeracy and functional maths, and also the foundations for A-Level Maths. Perhaps, like English, there ought to be two GCSEs in maths. One that tests numeracy and applications, linking to other subjects and drawing on the style of Core Maths, then another that develops the foundations for further study at A-Level.

As I write this, I can picture the exasperation of Key Stage 4 teachers at the thought of another reform, aimed at something as nebulous as culture, and particularly given the dearth of maths teachers available to implement this. Timing is important. It will be another 13 years before pre-school children are selecting their post-16 options. For those students to be desiring of, and prepared for, post-16 maths, we need an ambitious, long-range plan that builds in time for consultation and implementation.

But let us imagine that we have successfully shifted attitudes: students now believe themselves capable of learning maths, and both they and the educators who advise them recognise the importance of maths for their life choices, employability and navigation of an increasingly technological world. More students than ever enjoy learning maths. What should the curriculum offer be?

The post-16 experience of maths must be positive. While there is a suite of existing qualifications that will suit most students, and which are deserving of better promotion, there is an opportunity to do something innovative. For example, a 'History of Maths' course for historians, to develop understanding of maths in the context in which it was developed; or a coding course, to develop vital skills while exploring real-world data; or a course in statistics and modelling for economists, geographers or sociologists, to better prepare them for university study.

There are lots of details that need to be carefully thought through. If a course already contains a significant mathematical component, is that enough? How does this fit with those studying for apprenticeships or T-Levels? Does everyone need to study for a maths qualification, or might this be delivered like the pastoral curriculum; essential but not recognised with a qualification?

Whichever way it is implemented, it must be of a high quality, and done so without devaluing the arts, languages or humanities. Neither should it discourage students from taking up the excellent courses that are already available to them, including Core Maths, AS-Level Maths and A-Level Maths and Further Maths.

13. Education Endowment Foundation Teaching and Learning Toolkit. (2021) "Mastery learning". Education Endowment Foundation. <https://educationendowmentfoundation.org.uk/education-evidence/teaching-learning-toolkit/mastery-learning>

How GCSE resits lie within this new landscape also needs careful consideration. We know that having GCSE Maths opens up doors and empowers students with choices.¹⁴ We also know that at age 16, disadvantaged students are less likely to achieve a good pass in GCSE Maths than their peers, but are more likely to achieve one by age 18 than the more advantaged students who also resit.¹⁵ This impact on social mobility is significant, unlocking greater choice for disadvantaged students than any other post-16 programme of study.¹⁶

Importantly, the drive to have everyone learn some maths must not lead to existing post-16 qualifications being replaced with watered-down courses, delivered online and of less value. My further education colleagues ascribe the success of GCSE resits to putting their best maths teachers with these classes, who then instil a sense of belief in students that they can and will succeed. More broadly, the government should review post-16 funding to enable colleges to attract excellent practitioners to deliver these programmes.¹⁷

Specialist maths schools are well placed to extol the virtues of studying post-16 maths and, through our outreach programmes, to encourage greater uptake amongst students. Exeter Mathematics School, which I am honoured to lead, was one of the first, opening its doors in 2014, along with King's Maths School. Since then, four more maths schools have opened and plans exist for a network of 11 schools, each serving different regions in England.

U-Maths is a new initiative being set up by the specialist maths schools and their partner universities, to support one another, to aid collaboration and to widen participation in the mathematical sciences. While there are pitfalls to avoid, the principle that more students should study post-16 maths is a good one, which U-Maths is keen to support.

We have a mission as maths schools to support access to maths from under-represented groups. We recognise that increasing the prevalence of high quality teaching is one way to achieve this. With the right funding, we could form a network of regional teaching schools, upskilling non-specialists and supporting pedagogy for Further Maths teachers.

The maths to 18 agenda opens the possibility for innovative curriculum developments, enhancing the suite of existing qualifications and improving the life chances of students. To be a success, work is needed to develop clear messaging about the reasons why post-16 maths is important, to lay the foundations with pre-school children and their families, and to properly review the pre-16 maths curriculum and its delivery, prior to implementing changes at post-16.

The network of specialist maths schools is well-placed to help. If culture and curriculum can be delivered well, far from being forced to study maths, students will be clamouring to do so.

14. Smith, A. (2017) op. cit.

15. Department for Education. (2023), "Level 2 and 3 attainment age 16 to 25, academic year 2021/2022". <https://explore-education-statistics.service.gov.uk/find-statistics/level-2-and-3-attainment-by-young-people-aged-19>

16. Hodge, L., et al. (2021) "GCSE attainment and lifetime earnings". Government Social Research. https://assets.publishing.service.gov.uk/government/uploads/system/uploads/attachment_data/file/993202/GCSE_Attainment_and_Lifetime_Earnings_PDF3A.pdf

17. OECD. (2021) "Education at a Glance 2021". https://www.oecd-ilibrary.org/education/education-at-a-glance-2021_b35a14e5-en



Gareth Stevens

Chief Executive, Inspiration Trust

SEGMENTING THE MATHS COHORT

Conceptualising four pathways for maths to 18 can help policymakers, school leaders and students themselves to consider the different routes and options. Three of these pathways already exist within the education system, and can be built upon. The fourth, a new applied numeracy course, would need to be developed. For maths to 18 to be a success, the system should offer more incentives for maths teachers, alongside deploying technological solutions and online teaching.

The necessity of mathematics study for all students until they are 18 cannot be overstated, given the myriad advantages it offers. Mathematics nurtures critical thinking and skills for resolving problems that find application in various contexts beyond educational boundaries. It serves as a scaffold for organised cognition, assisting students in unravelling intricate issues and enabling judicious decision-making.

The pursuit of mathematics enhances numeracy, a critical ability in today's information-centric world. It bolsters analysis and accuracy, skills prized in a wide range of sectors, including technology, scientific research and finance. Additionally, it nurtures an understanding of the inherent interconnected patterns in our surroundings. Competence in mathematics paves the way to improved financial awareness, preparing students to handle their personal finances efficiently.

It is no surprise, given this, that the Prime Minister has expressed interest in engaging with maths to 18 for all students. To help think about how this works, I propose a way of conceptualising the cohort, while acknowledging the challenges that our current system may encounter during the implementation.

Post-16 maths should be divided into four principal pathways, which align with students' respective starting points, following their Level 2 maths qualifications (usually GCSE).

1. **A-Level Maths and Further Maths:** Designed for students who have achieved grade 7+ in their Level 2 maths qualifications.
2. **AS-Level Core Maths:** Tailored for students who have achieved grade 5 or 6 in their Level 2 maths qualifications. Students would study this course for two years, in tandem with other A-Levels. Students with level 7+ and taking A-Levels would be required to follow this path if they did not want to study A-Level Maths.
3. **A new applied numeracy course:** Targeted at students who have achieved grade 4 or 5 in their Level 2 maths qualifications, and are undertaking Level 2 or Level 3 vocational qualifications at a college. This course would concentrate on how maths connects to everyday life, with a special focus on personal finances.
4. **GCSE Maths resits:** This pathway is intended for any student who did not attain a passing grade in their Level 2 maths qualifications. These students would have the opportunity to resit the exam to improve their understanding and competence in maths, ensuring that they do not miss out on the many benefits of mathematical literacy.

This structure ensures an inclusive approach to maths education, catering to diverse student needs and abilities, while also promoting the importance of numeracy skills in various aspects of life.

Three of these pathways already exist within the education system, but to ensure success, we must put more weight on the strategies proposed. Specifically, GCSE Maths resits need attention. Too frequently they are perceived as secondary considerations. The responsibility for this must be raised to a higher level, guaranteeing that students receive optimal opportunities to achieve mathematical literacy. I would suggest including this figure in sixth form and college performance tables, not within the main body of key performance indicators, but on the front page.

Enacting a strategy that calls for systemic change, and a new qualification, inevitably poses challenges, not least the daunting task of staffing an increased number of maths lessons.

First and foremost, we must recognise the dire need to recruit more maths teachers. To accomplish this, the system should offer incentives for maths positions, ensuring that top talent is compensated comparably to other postgraduate positions in mathematics. This is embodied by the Mathematics Teacher Training Scholarships, an initiative funded by the Department for Education and managed by the Institute of Mathematics and its Applications, expressly designed to attract gifted mathematicians into teaching.

Second, aside from recruiting and retaining more maths teachers, there are other strategies that can be adopted to reduce the demand for such teachers. For instance, the Inspiration Trust is currently teaching GCSE Further Maths to 350 students through online delivery. These students would otherwise have been deprived of the opportunity to study GCSE Further Maths, as their schools did not offer the course.

The course is delivered by Sir Isaac Newton Sixth Form staff over two evenings per week (one Year 10, one Year 11). This method only requires two staff per 175 students; one to teach and another to respond to live queries. Despite some limitations, this method presents a viable way to offer courses that would otherwise be inaccessible for these motivated students. The advent of technological solutions such as this offers promising alternatives for course delivery and makes courses like GCSE Further Maths accessible to more students.

In essence, ensuring comprehensive maths education to age 18 is an endeavour that will involve overcoming significant challenges, but with concerted effort and creative strategies, it is an attainable and worthwhile goal.

T	S	Y	Z	E	S	D	C	O	R	Y	L
G	J	T	U	L	P	A	E	X	F	S	K
A	S	E	C	T	I	O	N	F	3	O	V
Q	E	O	H	F	X	K	S	B	F	N	C
S	R	L	B	J	D	Q	Y	G	K	P	M

Design Principles



Sherry Coutu CBE

Chair, Maths4Girls

EMPOWERING GIRLS AND YOUNG WOMEN IN MATHEMATICS

We know, and the 'Mathematics Pipeline in England' report commissioned by XTX Markets shows very clearly, that girls are more likely than boys to have poor engagement with maths, and less likely to progress to post-16 maths.¹⁸ Opening up maths to 18 requires additional focus on the root causes of girls' under-engagement with maths, or else it risks exacerbating these gaps. By contrast, creating a positive impact on girls' engagement and achievement in maths can pave the way for a more equitable and prosperous future. Emerging evidence, including from a programme that I chair, suggests transferable lessons that can be learned.

18. Noyes, A., et al. (2023) op. cit

The introduction of maths to 18 holds tremendous potential to reshape maths education, fostering inclusivity and equity. However, addressing the challenges that hinder girls' engagement and achievement in mathematics currently requires targeted interventions that are scalable and cost-effective. The under-representation of girls in post-16 maths impairs both individuals and the productivity of organisations employing them once they enter the workforce and our society. Without addressing this, or addressing it only through compulsion, the expansion of maths to 18 risks simply exacerbating, rather than solving, the current gender gaps in uptake, attainment and enjoyment.

Maths4Girls, a high-impact and award-winning initiative that I am privileged to chair, has worked with more than 2,000 schools and over 600,000 students to seek to understand and mitigate the root causes of girls' engagement with maths.¹⁹ The critical phase of education is age 11-16, as that is where so many perceptions of maths are formed. It is also where the building blocks of GCSE-level content are needed to provide a base for post-16 study. Maths4Girls specifically focuses on the 11-14 phase and brings together teacher-led interventions to address misconceptions with maths.

This experience has allowed for the identification of wider lessons which can be learned when focussing on girls' engagement with, and attainment in, maths. There are four key lessons.

First, confidence, and empowerment. To empower girls in maths, and in other areas where they are under-represented, creating an inclusive and supportive learning environment is crucial, allowing women to excel in their own terms. Early engagement plays a vital role, as research indicates that attitudes towards maths are often shaped in early childhood.²⁰ But such interventions also need 'topping up'. As girls grow up and progress through school, care needs to be taken that early-stage confidence does not fade away or get undermined.

Second, teacher confidence. As so many of the essays in this collection have noted, there is a clear link between the expertise and passion of the teacher, and their effectiveness at teaching maths. If teachers feel confident in their subject, and the ways in which that subject will open up paths in the future, then they will be able to address misperceptions and barriers which exist among students. Supporting teachers empowers them to become more effective at communicating the benefits and opportunities associated with maths education to girls. Through this, girls can envision themselves pursuing post-16 maths because they understand potential career paths and the impact that mathematics can have.

Third, and relatedly, role models. We know that people seeing 'people like them' can have an impact on aspirations.²¹ But teachers face time constraints and can struggle with the complexities of bringing guest speakers into the classroom. Specialist services, including the Careers and Enterprise Company, can help. Increasingly, so too can specialist online platforms, which connect girls studying maths with highly-rated volunteers who are trained to highlight the relevance and applicability of mathematics across various fields. Sustained engagement enhances girls' motivation and success in the subject. Equally importantly, technology reduces the time and cost barriers to engagement. Such barriers often privilege those schools and communities that already have strong networks. Technology can help to address the inequalities in schools that do not have access to parents with high levels of education or professional jobs.

Fourth, careers guidance. Interventions for 11-14 year olds often focus on skills or experiences that are either directly applicable to the labour market, or that help young people to embark on a career in the near future. However, existing research suggests that the quality and coverage of careers education in the secondary phase is variable. For example, a study drawing on large-scale surveys of students aged 15 and 16, and longitudinal qualitative interviews with 10-16 year olds, found that over a third of students in their final year of secondary school had not received any form of careers education. Of these, only a little over half were satisfied with the quality of careers education that they received, leading to calls from the same students for more and better provision. The report concluded that:

*Careers education is not currently reaching those most in need of it. Girls, minority ethnic, working-class, lower-attaining and students who are unsure of their aspirations or who plan to leave education post-16 are all significantly less likely to report receiving careers education.*²²

19. Maths4Girls. (2023) <https://m4g.founders4schools.org.uk/>

20. National Association for the Education of Young Children. (2002) "Early Childhood Mathematics: Promoting Good Beginnings". <https://www.naeyc.org/sites/default/files/globally-shared/downloads/PDFs/resources/position-statements/psmath.pdf>

21. González-Pérez, M., et al. (2020) "Girls in STEM: Is It a Female Role-Model Thing?". *Frontiers in Psychology* 11. <https://www.frontiersin.org/articles/10.3389/fpsyg.2020.02204/full>

22. Archer, L., et al. (2020) "ASPIRES 2: Young people's science and career aspirations, aged 10-19". UCL Institute of Education. https://discovery.ucl.ac.uk/id/eprint/10092041/15/Moote_9538%20UCL%20Aspires%20%20report%20full%20online%20version.pdf

The transformative power of Maths4Girls is evident through its widespread adoption and remarkable impact. A single Maths4Girls role model encounter has been shown to increase girls' awareness of maths by 15 percentage points, elevate their plans to pursue post-16 maths by nine percentage points and enhance their confidence in the subject by seven percentage points.

The adoption of maths to 18 presents a unique opportunity to reshape maths education and foster a more inclusive learning environment. By harnessing the power of technology and focusing on practical barriers that often underpin girls' under-engagement with maths, it is possible to overcome the barriers they face and foster their future success.

Maths to 18 can, and indeed must, create a transformative impact on girls' engagement and achievement in maths, paving the way for a more equitable and prosperous future.



Simon Coyle

Head of Philanthropy, XTX Markets

INCREASING A-LEVEL MATHS TAKEUP AMONG HIGH-ATTAINING 16 YEAR OLDS

As an organisation that supports maths excellence, XTX Markets believes that any maths to 18 policy should maximise takeup of advanced Level 3 maths qualifications among high-attaining 16 year olds. One risk with a new qualification is that it could 'cannibalise' from students who are already taking A-Level Maths, as well as students who might otherwise choose A-Level Maths if studying maths to 18 was a requirement. Any new policy should be designed to incentivise and support more students to choose advanced Level 3 maths qualifications, which will enable them to progress to degrees in maths and other quantitative subjects.

XTX Markets partners with charities, schools and universities to support more young people to progress to maths degrees and PhDs, and to become highly skilled workers. We believe it is crucial that significantly more people are supported to excel in mathematics, which will support discovery, innovation and the jobs of the future, including an expansion of the teacher workforce.

Around 550,000 18 year olds in England completed their studies in 2021/22. This included roughly 90,000 A-Level and 9,000 AS-Level Maths entries, and roughly 16,000 A-Level and 4,000 AS-Level Further Maths entries. It also included an estimated 12,000 Core Maths entries. The largest groups, however, were the roughly 150,000 students who retook GCSE Maths and the roughly 250,000 students who did not take any maths qualifications at all.

Given our wider goals around maths excellence, XTX Markets supports the principle that all students should study some form of maths to 18. We believe that any policy mandating this should aim to maximise the number of students who take advanced Level 3 maths qualifications, enabling them to progress to degrees in maths and other quantitative subjects.

Data from *The Maths Pipeline in England*, a longitudinal study by the University of Nottingham, highlights five groups of students: (i) roughly 8,000 who attained grade 7+ in GCSE Maths and studied both A-Level Maths and Further Maths; (ii) roughly 40,000 who attained grade 7+ and studied A-Level Maths; (iii) roughly 4,000 who attained grade 6 or below and studied A-Level Maths; (iv) roughly 45,000 who attained grade 7+ and did not take A-Level Maths; and (v) roughly 380,000 who attained grade 6 or below and did not take A-Level Maths.

This essay focuses on groups (ii), (iii) and (iv). These are the roughly 45,000 students who studied A-Level Maths but not Further Maths, and the roughly 45,000 who attained top grades in GCSE Maths but chose not to take A-Level Maths. How could a new maths to 18 policy maximise take-up of advanced maths courses among these 90,000 students?

At present, the main Level 3 options for these students are A-Level Maths, AS-Level Maths and Core Maths. Only a small proportion take AS-Level Maths, which is no longer convertible to A-Level, or Core Maths, a standalone qualification. In a world where all students must study some form of maths to 18, the main issue affecting these target students is whether to add a new Level 3 qualification, pitched between A-Level Maths and those other courses.

I agree with the widely held view that Core Maths is 'a good thing' that should certainly be part of the maths to 18 offer. Although the different Core Maths specifications are not always equivalent, generally speaking it fulfils an important role. However, as someone who is currently taking A-Level Maths – 20 years late, regrettably – it seems to me that there are fairly large gaps between parts of A-Level Maths and Core Maths, and some parts that are missing altogether.

If we want to maximise take-up of advanced Level 3 Maths qualifications, and enable more students to choose degrees in maths and other quantitative subjects, I have two worries regarding Core Maths. First, it does not cover some topics like calculus that are crucial for quantitative degrees, such as economics. Second, there is no convertibility to A-Level Maths, so students who thrive in Core Maths cannot easily 'trade up' to A-Level.

In a world where all students must study maths to 18, high-attaining students that do not take A-Level Maths and instead choose Core Maths would be showing their hand, i.e. they would be actively choosing a less deep course option.

Given that many high-attaining students tend to aspire to high-tariff universities, in this new world we might expect a large number to now choose A-Level Maths (stronger signal to universities) rather than Core Maths (less strong).

Conversely, if a new Level 3 qualification was pitched between A-Level Maths and the other courses such as Core Maths, students might see this as a 'goldilocks' option. This would likely be popular with high-attaining students not currently choosing A-Level Maths, i.e. group (iv). It may also lead to cannibalisation from A-Level Maths, i.e. groups (ii) and (iii).

As such, I would be cautious about creating any new qualification that might fail to maximise, or might reduce, entries for A-Level Maths, and thus limit progression to degrees in maths and other quantitative subjects.

I am on the fence as to whether having convertibility from other courses to A-Level Maths would help or hinder. It seems likely that a convertible 'goldilocks' option would be popular as a hedging choice, which would mean a net increase in the number of students at least *joining* the pathway to A-Level Maths. However, there may be too little 'trading up' to create a net increase in A-Level Maths completions overall.

So, given our aim of maximising takeup of advanced Level 3 maths qualifications, perhaps the best option is to maintain the gaps between A-Level Maths and the other courses. In the world where students must study some maths to 18, they would be forced to show their hand, and it seems plausible that a significant number of group (iv) would be comfortable with and excel in A-Level Maths.

It is worth emphasising here that, if maths does become a compulsory subject to age 18, it would be an exceptional case in the English school system. Thus, it might warrant special consideration, such as reintroducing the convertibility from AS-Level to A-Level Maths.

Another option, which is more of a wildcard, could be to introduce a baccalaureate-style qualification, embedding maths to 18 within wider reforms to post-16 education. This would take longer and face significant implementation challenges. However, both the Conservatives and Labour have an indicated openness to a baccalaureate-style qualification, so let me muse on this a little.

Imagine that we have two pathways in maths from 16-18, one academic and one functional. Both pathways are split into standard and higher streams, with each of these representing one course choice.

The functional pathway focuses on applications of maths and preparation for technical roles. The standard stream replaces GCSE retakes with a new qualification focused on applications. The higher stream ranges from parts of Core Maths through to further applied maths content, such as mechanics and data science.

The academic pathway focuses on fundamental maths, preparing for degrees in maths and other quantitative subjects. The standard stream ranges from parts of Core Maths through to A-Level Maths. The higher stream ranges from AS-Level Maths through to approximately 60% of A-Level Further Maths.

Crucially, the academic pathway would allow students to complete different qualifications; adjusting and confirming their choices over two years of study. AS-Level Maths would yield 60 UCAS points, A-Level Maths 120 points, and A-Level Further Maths 180 points. This would of course represent special treatment for maths against all other subjects, but as noted this may be justifiable given its compulsory status.

	Standard Stream	Higher Stream
Functional Pathway	Level 2 – Replacement for GCSE retakes, focus on applications.	Level 3 – Combination of Core Maths and applied maths (e.g. mechanics).
Academic Pathway	<u>60-120 UCAS points</u> Core Maths AS-Level Maths A-Level Maths	<u>60-180 UCAS points</u> AS-Level Maths A-Level Maths A-Level Further Maths (~60%)

One issue with the current system is that it *de facto* requires students who intend to pursue a maths degree to choose A-Level Maths and Further Maths, putting two of their three eggs in one basket. By permitting students to commit only one A-Level choice to maths, the above model potentially opens up the subject to students who would be otherwise reluctant to use two choices on A-Level Maths and Further Maths.

There is some evidence that 'over-specialisation' deters female students in England from continuing with maths to 18. Female and male students attain basically equal rates of grade 7+ in GCSE Maths, but far fewer female students choose A-Level Maths or A-Level Further Maths, or go on to maths degrees. This pattern is less pronounced in Scotland, where Level 3 qualifications and entry expectations for maths degrees are arguably more flexible. For example, both Edinburgh and Glasgow universities report having an undergraduate intake in maths that is 43% female and 57% male.

There is an appeal to creating a new 'goldilocks' qualification, pitched between A-Level Maths and other Level 3 courses. However, if we want to maximise the number of students who complete advanced Level 3 maths qualifications, this risks cannibalising the potential A-Level Maths cohort.

A bold option would be to add no new qualification, forcing high-attaining students to show their hand on A-Level Maths. An even bolder option would be a baccalaureate-style qualification, offering multiple pathways within one subject choice.



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THE ROLE OF CORE MATHS

In the medium-term, a system of maths to 18 should be part of a wider change to post-16 education, such as that imagined through a British Baccalaureate. However, such change will take time. In the meantime, Core Maths offers us a way to deliver a 'broad and balanced' education to thousands of 16-18 year olds not studying A-Levels. Core Maths has strong support from universities, schools and colleges. With further support, Core Maths could be offered to far more than the 5-10% of students who could benefit from it and are currently taking it.

In outlining his vision for maths to 18, Prime Minister Rishi Sunak argued that our future economic growth depends on maximising the potential of all young people, and that we should move towards a model where all children study some form of maths to 18:

In a world where data is everywhere and statistics underpin every job, letting our children out into that world without those skills, is letting our children down.

I could not agree more.

The importance of increasing the mathematical, quantitative and data skills of young people and adults has never been greater, and the breadth and depth of knowledge and accompanying skills that are needed is vast and ever-growing.

The union of all the 'maths' implicit in the PM's statement includes the mathematical, quantitative and data skills that the UK will need in the future. However, it does not mean that every child should study everything that might be available in any subsequent post-16 offer. It also does not mean, as the PM made clear, that every child should study A-Level Maths.

A post-16 British Baccalaureate, such as the one proposed by the Times Education Commission, would be a sensible way forward to achieve the PM's ambitions.²³ In any British Baccalaureate, we must retain the opportunity to study much of the current content and associated skills in the reformed AS-Level and A-Level courses, which have been well received by higher education.

These qualifications bring a welcome focus on mathematical argument, language, proof, problem-solving and modelling, as well as the use of technology and the use of data in statistics. This provides much of the vital underpinning in post-16 studies for scientists, mathematicians, engineers and data scientists, who will ultimately address our global challenges.

On top of this, in a reformed pre-16 framework, like the one advocated by the Times Education Commission, there must be a strong emphasis on data and statistical literacy, and this should permeate the curriculum. It most certainly does not in our current pre-16 curriculum, nor in our current GCSE Maths curriculum. Strengthening these emphases would enable all students to benefit fully from a broadened post-16 curriculum.

However, any such reforms to introduce a British Baccalaureate and to reform the 11-16 curriculum will not be easy, and will take time. In the meantime, much more needs to be done to support the core premises that they offer.

Foremost among them is a focus on a 'broad and balanced curriculum', a key thrust of the Royal Society's work on maths education through its Advisory Committee on Mathematics Education (ACME) since the launch of its Vision in 2014.²⁴ Within the Royal Society's plans for maths education, Core Maths was an integral part. In response to the PM's new announcement, the President of the Royal Society, Prof Sir Adrian Smith wrote:

*The introduction of Core Maths, as an alternative qualification to maths A-Levels has been a popular and positive change. But more still needs to be done to make such courses, and mathematics skills in general, widely available and appealing to students.*²⁵

In any such reforms to post-16 maths, whether within a broader baccalaureate system or not, we must also retain the opportunity to build on the great strides made in Core Maths.

What is important is what Core Maths represents. It focuses on: deepening competence in the selection and use of mathematical methods and techniques; developing confidence in representing and analysing authentic situations mathematically; applying mathematics to address related questions and issues; and building skills in mathematical thinking, reasoning and communication.

23. The Times Education Commission. (2022) "Bringing out the best: how to transform education and unleash the potential of every child". <https://nuk-tnl-editorial-prod-staticassets.s3.amazonaws.com/2022/education-commission/Times%20Education%20Commission%20final%20report.pdf>

24. Royal Society Advisory Committee on Mathematics Education. (2022), "A broad and balanced curriculum". <https://royalsociety.org/topics-policy/projects/vision/science-to-18/>

25. Royal Society. (2023) "Royal Society responds to expert-led review of Maths education to 18". <https://royalsociety.org/news/2023/04/rishi-sunak-mathematics-speech-response/>

Core Maths, either as now, or as it might evolve in a British Baccalaureate, must play an even greater role in the future. It focuses on developing fluency and confidence in using and applying mathematical and statistics skills to address authentic problems, drawn from study, work and life, with a strong emphasis on contextualised problem-solving. This is precisely what is needed to improve the mathematical and quantitative skills of young people who are not studying the mathematics that is currently found in A-Level Maths.

While much has already been done to work with stakeholders, particularly with universities across a wide range of undergraduate programmes, more must be done on this front, and with a wider range of stakeholders, in order to significantly increase the number of students taking Core Maths.

Many universities have expressed their support for Core Maths. The experience on the ground is that higher education staff are impressed by the nature and content of the qualification, with its emphasis on contextualised problem-solving.²⁶ Many schools and colleges have also embraced Core Maths, providing students with the opportunities they deserve.

Much less is known, however, about employers' knowledge and understanding of Core Maths. Therefore, it is a priority for more engagement to take place with businesses, employers and industry to improve this situation.

In Smith's insightful 2017 review of post-16 mathematics, the first of his 18 recommendations was that:

*The Department for Education should seek to ensure that schools and colleges are able to offer all students on academic routes and potentially students on other Level 3 programmes access to a core maths qualification.*²⁷

Everyone who achieves a pass in GCSE Maths in our current framework, but who chooses not to take A-Level Maths, should have opportunity to benefit from Core Maths. Yet in the six years since that report, the number of students that are taking Core Maths is still only at most 5-10% of those who could benefit from doing so.²⁸

Core Maths is mission critical to the UK, not least for those young people who could benefit from taking it. Now is the time for all stakeholders to support the ambition of many more students taking Core Maths, and to lobby for the three important changes:

1. Significant additional funding for all students taking Core Maths in schools and colleges.
2. Universities being more specific and strengthening their signalling of the benefits of Core Maths.
3. Employers making clear that Core Maths provides the mathematical, quantitative and problem-solving skills needed for many roles in their organisations.

We have much to gain from realising the Prime Minister's vision, including universities and their students, employers and their employees, and ultimately UK society. Core Maths offers us a route to get there.

26. See, for example, those gathered here: http://www.personal.reading.ac.uk/~smsglais/University_Statements_on_Core_Maths.pdf

27. Smith, A. (2017) op. cit.

28. Department for Education. (2023) "Post-16 maths participation for pupils ending KS4 in 2018/19". https://assets.publishing.service.gov.uk/government/uploads/system/uploads/attachment_data/file/1132372/Post_16_maths_participation_ad_hoc_statistics.pdf; and

Hodgen, J., et al. (2014) "Mathematical transitions: a report on the mathematical and statistical needs of students undertaking undergraduate studies in various disciplines". Higher Education Academy. https://mei.org.uk/app/uploads/2021/08/hea_mathematical-transitions_webv2.pdf



Sarah Waite

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MATHS TO 18: THE ROLE OF FURTHER EDUCATION

Improvement in maths attainment has a significant effect on lifetime earnings. That is why, since 2014, young people who do not attain a good pass in GCSE Maths at age 16 continue to study towards it, many in further education colleges. More broadly, the FE sector will be a critical part of any policy of maths to 18, educating hundreds of thousands of young people a year. Neuroscience is clear that age 16 is not too late, there is a huge opportunity to embed maths excellence in these young people. Doing so requires action on FE curricula and support for FE lecturers.

Maths to 18 is not a new idea. Most of the developed nations in the OECD ensure that young people study some form of maths until adulthood. Where countries have experimented with dropping this policy, for example France in 2019, politicians have quickly reversed course. Just two years later, President Macron sacked his Education Minister and restored maths as an obligatory core subject in the baccalauréat.

It is not even a new idea within the UK. In 2011, the Secretary of State for Education, Michael Gove, gave a speech setting out that, within the next decade, it should be a goal for most students to study maths to age 18.²⁹ Maths and English to 18 was included as a commitment in the Labour Party manifesto in 2015.³⁰

Why are developed nations so keen on more maths? Primarily because research, both at home and internationally, strongly suggests that there is a link between studying maths for longer and future earnings. The effect of a one-grade improvement at GCSE on lifetime earnings is stronger for maths than any other subject. While the effect was strongest at the pass grade border, there is a financial benefit to moving up at any grade boundary in maths.³¹

This is why ministers in England recognise that not securing a pass in GCSE Maths at age 16 has a lifelong impact on the opportunities available for young people. Without passes in GCSE Maths and English, a young person is nine times more likely to be 'not in education employment or training' (NEET) by age 18.³²

Missing these qualifications is the biggest factor in becoming NEET, accounting for more than socioeconomic background or having a special educational need and/or disability.³³ Without these qualifications, opportunities for further and higher study are dramatically reduced. Fewer than 12% of young people without passes in GCSE Maths and English at age 16 are studying A-Level qualifications the following year. They comprise just 20% of starters on Level 3 apprenticeships.³⁴

That is why, in 2014, our education system changed to ensure that students who do not attain a good pass in GCSE Maths and English at age 16 continue to study some form of maths in post-16 education. Since the introduction of this policy, the number of young people who study maths in post-16 education has increased dramatically.

While pass rates remain too low overall, standards have risen. Two decades ago, fewer than 7.8% of young people who had not passed GCSE Maths at age 16 had gone on to achieve this by age 19. Today, that figure is four times higher. In the last academic year, a higher proportion of young people from disadvantaged backgrounds passed GCSE Maths in post-16 education than their non-disadvantaged peers.³⁵ This bucks the longstanding attainment gap observed earlier throughout the education system.

Despite this change, around half of all 16-19 year olds do not study any form of maths at all.³⁶ There is also a perception among many that age 16 is too late, and that students, especially those who may have struggled in school up to then, should not engage further with maths.

Here is why I disagree.

29. DfE. (2011) op. cit.

30. The Labour Party (2015) "Britain can be better: The Labour Party Manifesto 2015". <https://manifesto.deryn.co.uk/wp-content/uploads/2021/04/BritainCanBeBetter-TheLabourPartyManifesto2015.pdf>

31. Hodge, L., et al. (2021) op. cit.

32. Lupton, R., et al. (2021) "Moving on from initial GCSE 'failure': Post-16 transitions for 'lower attainers' and why the English education system must do better". Nuffield Foundation. <https://www.nuffieldfoundation.org/wp-content/uploads/2021/02/Post16-transitions-for-lower-attainers-Final-report.pdf>

33. Brooks, R. (2014) "Out of sight: how we lost track of thousands of NEETs, and how we can transform their prospects". The Fabian Society. https://www.impetus.org.uk/assets/publications/Report/Out-of-Sight_Fabians_Impetus-PEF_12_2014.pdf

34. Lupton, R., et a. (2021) op. cit.; and DfE, 'Apprenticeships and traineeships'.

35. Gov.uk. (2023) "Level 2 and 3 attainment age 16 to 25". <https://explore-education-statistics.service.gov.uk/find-statistics/level-2-and-3-attainment-by-young-people-aged-19>

36. Gov.uk. (2023) "Prime Minister sets ambition of Maths to 18 in speech". <https://www.gov.uk/government/news/primeminister-sets-ambition-of-maths-to-18-in-speech#:~:text=The%20Prime%20Minister%20will%20commit,study%20any%20maths%20at%20all.>

The latest evidence in neuroscience challenges the longstanding consensus that the early years are the only opportunity to maximise cognitive potential. Neuroscientists have now proven that there is another 'window of opportunity' in which the brain is particularly receptive to learning certain skills or knowledge, and that this takes place in late adolescence and early adulthood.³⁷ This period is associated with increased neuroplasticity; the brain's ability to change and adapt.

Studies show that adolescents benefit from targeted education and training interventions.³⁸ This window is particularly important for developing higher-order cognitive skills, social and emotional skills, and specialised knowledge in areas like mathematics, science and the arts.

We now know that, not only are young people's brains not fixed at age 16, but that post-16 education presents a critical moment in education. This is something that I get to witness first hand through the work of Get Further, the charity that I lead. Every year, we partner with colleges and sixth forms to match top tutors with students aged 16 or older, who have not yet achieved a good pass in GCSE Maths or English. Our tutors support students to build their literacy and numeracy skills, as well as their confidence, helping them gain these gateway qualifications, which unlocks opportunities for further study and work.

We see what is possible when young people who did not reach the expected standard at school receive targeted support in post-16 education. After a term of tutoring with Get Further, pass rates are twice the national average and progress is seven times the national average.³⁹

Despite the latest science, the idea that age 16 is 'too late' persists. Decisions on education funding are reflective of this. When faced with a decision over which part of the education budget to cut in 2013, Gove concluded that reducing funding for 18 year olds was the "least detrimental" option.⁴⁰ Today, per student funding falls at age 16, and funding to support students from disadvantaged backgrounds also nosedives.

This has a far-reaching and often disastrous effect on further education. At present, 16-19 year olds are funded for only half the timetabled hours as their peers in other leading economies.⁴¹ There is less teaching time and fewer resources available for young people, just at the time when there is a critical window of opportunity for their development.

Within this context, extending maths to 18 poses a real opportunity. A policy of additional maths teaching will need to include an offer for the hundreds of thousands of young people in FE colleges. This should address three areas: curriculum, recruitment and funding.

Most of the maths courses currently delivered in FE colleges are GCSE or Functional Skills. Extending maths to 18 would require FE colleges to deliver additional maths qualifications, including options that fall between GCSE and A-Level, in order to support students who attained grade 4 or 5 in GCSE Maths. This might be Core Maths, something similar or something entirely new.

37. Robson, D. (2017) "There is a second "window of opportunity" for learning in late adolescence and early adulthood". <https://psychpstuff.wordpress.com/2017/04/25/there-is-a-second-window-of-opportunity-for-learning-in-late-adolescence-and-early-adulthood/>

38. Knoll, L. J., et al. (2016) "A window of opportunity for cognitive training in adolescence". *Psychological Science* 27 (12) pp1620-1631. <https://psychpstuff.wordpress.com/2017/04/25/there-is-a-second-window-of-opportunity-for-learning-in-late-adolescence-and-early-adulthood/>

39. Get Further. (2023) "Helping Students Get Further: Our Annual Impact Report 2021/22". <https://getfurther.org.uk/wp-content/uploads/2023/06/GET-FURTHER-GCSE-Impact-Report-21-22.pdf>

40. Linford, N. (2014) "Colleges hit by 18-year-old funding rate cut far worse than school sixth forms, government assessment reveals". <https://feweek.co.uk/colleges-hit-by-18-year-old-funding-rate-cut-far-worse-than-school-sixth-forms-government-assessment-reveals/>

41. Hodgson, A., and Spours, K. (2016) "Tuition time in upper secondary education (16-19): Comparing six national education systems". UCL Institute of Education. <https://sfcawebsite.s3.amazonaws.com/uploads/document/Tuition-time-in-upper-secondary-education-2016.pdf?t=1545389697>

FE colleges will also need to be supported to overcome workforce issues, which are more acute in FE than in schools. Incentives to recruit more maths teachers to address shortages are only offered to schools and not colleges. Teachers in FE colleges are paid on average £7,000 a year less than their peers in schools.⁴² Not only does this create challenges for attraction, but it also leads to high levels of churn, with FE maths teachers easily tempted away to schools.

Alongside pulling existing levers, like wages, to attract more maths teachers and retain existing ones, the Department for Education will have to get creative to support teacher recruitment into FE. The DfE is now predicting a larger drop in primary-age students from next year onwards than previously thought.⁴³ Fewer primary-age students means fewer primary teacher vacancies. The DfE should consider free A-Level Maths courses (for those without it) and professional development for primary teachers who transition to post-16 education. The DfE will also need to consider strategies to upskill the existing FE teaching workforce, many of whom do not hold a maths qualification beyond GCSE.⁴⁴

Maths to 18 is a positive leap towards an education system that takes a holistic approach to maths. For all learners, post-16 education should mark the transition to the next stage of their journey in maths, as is the case in most other developed nations. A system in which all learners study some form of maths to 18 would have a substantial effect on removing any stigma associated with continuing to study GCSE Maths after age 16.

While they are not its intended focus, this policy should also improve the delivery of maths education for students with lower prior attainment. Currently, many FE colleges treat GCSE retakes as a nine-month course within one academic year. Maths to 18 would encourage those colleges to treat the GCSE as a two-year maths course, as it is designed. In this context, the principles of mastery teaching could be better embedded across FE maths programmes. In turn, this would better support young people who have a greater distance to travel to reach a good pass in GCSE Maths.

Maths to 18 offers the chance to set a vision for 16-18 maths as a true continuation of study, building on the level of knowledge and skills that young people have reached during their time in school. If actions are taken to enable this vision to be realised, the impact on all young people who would benefit from continuing to build their skills in mathematics would be far-reaching.

42. Sibieta, L., and Tahir, I. (2023) "What has happened to college teacher pay in England?". Institute for Fiscal Studies. <https://ifs.org.uk/sites/default/files/2023-03/What-has-happened-to-college-teacher-pay-in-England.pdf>

43. Julius, J. (2022) "Falling pupil numbers: What does this mean for the education system?". National Foundation for Economic Research. <https://www.nfer.ac.uk/news-events/nfer-blogs/falling-pupil-numbers-what-does-this-mean-for-the-education-system/#:~:text=For%20example%2C%20a%20school%20seeing,a%20large%20amount%20of%20funding.>

44. Frontier Economics. (2014) "The qualifications of English and mathematics teachers". The Education & Training Foundation. <https://www.et-foundation.co.uk/wp-content/uploads/2014/09/RPT-Survey-v4.pdf>

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Workforce



Prof Catherine Hobbs

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THE IMPORTANCE OF PROVIDING ROUTES TO MATHEMATICAL SCIENCES DEGREES

Maths to 18 does not mean A-Level Maths for all. Yet there are reasons that a larger proportion of the 16-18 cohort should have the opportunity to take A-Level Maths, and that they should have access to high-quality teaching and support to enable them to succeed. Doing so will provide a route into maths at undergraduate level and beyond, and a larger pool of potential maths teachers. At the same time, we must be careful not to narrow the pipeline at degree level by insisting, as universities increasingly are, that entrants must have attained grade A or even an A* in A-Level Maths.

When the maths to 18 policy was launched in January this year, many commentators and celebrities who assumed that the policy meant that A-Level Maths should be compulsory for all took to social media to express their outrage. Clearly, A-Level Maths for all was not the goal. It is neither feasible nor desirable to force all 16-18 year olds to take a course that they may have neither the prior attainment for, interest in, nor the need for in their further studies or careers.

Nevertheless, there is a strong argument that a larger proportion of the 16-18 cohort should have the opportunity to take A-Level Maths than is currently the case, and that they should also have access to high-quality teaching and support to enable them to succeed. I argue here for a setting a more ambitious target for the proportion of 16-18 year olds taking and passing A-Level Maths.

Many people are aware that maths is currently the most popular A-Level overall, having taken the top spot from English a few years ago. However, they may be less aware that, even so, only about 30% of the A-Level cohort take maths, and that the distribution is not even across all splits of the cohort. For example, about 40% of A-Level Maths students are female and 60% are male.⁴⁵ Access to A-Level Maths is limited in some areas, with eight Parliamentary constituencies reportedly offering no provision at all (Bolton West; Bury South; Stalybridge and Hyde; Barnsley East; Blackpool South; Bolsover; Dudley South; Houghton and Sunderland South).⁴⁶

We may ask, why it is important for more students in the 16-18 cohort to have the option to take A-Level Maths, and to have the support to reach their full potential? The issue is that it is an essential preparation for many STEM degrees and, in turn, to teacher supply. Access to many of these degrees requires the achievement of a very high grade.

At this point it is worth noting that A Level-Maths is not the only route to providing the mathematical skills needed for degrees in numerate disciplines. As Prof Paul Glaister writes in his essay, Level 3 qualifications in Core Maths were introduced for first teaching in 2014, and provide a really strong preparation for many degrees with a mathematical or quantitative element, including degrees in the social sciences that need mathematical thinking and practical skills. Unfortunately, a dearth of provision means that many students are unable to benefit from the excellent preparation that Core Maths provides. An increase in this provision is needed.

For A-Level Maths, an obvious point is that it is an essential requirement for all mathematical sciences undergraduate courses. The number of students entering mathematical sciences degrees has been in decline over the last 10 years, with an intake of around 6,300 UK-domiciled students in 2021 (down 9% since 2011). In contrast, computer science and engineering subject areas have seen significant increases over this period, with around 25,000 UK-domiciled students studying engineering (up 27% since 2011) and 24,500 studying computer science (up 34% since 2011).

These numbers are of course a positive. We know that the UK needs more qualified engineers and computer scientists. But mathematical scientists also support the UK's growing technical economy, as well as its strong financial sector, and have excellent graduate employment outcomes.

Mathematical sciences degrees are also the 'gold standard' for entry to maths teaching. It is well known that there has been a significant shortfall in the recruitment and training of new maths teachers over many years, and that there are currently very high vacancy rates. Increasing the supply of qualified maths teachers is the key to unlocking a large part of the maths to 18 agenda. This lack of maths teachers, ideally well-qualified ones, has been highlighted as a key barrier to delivering on maths to 18, including in several other essays in this collection.

45. Gov.uk. (2022) "'Entries and Results - A level and AS by subject and student characteristics (single academic year)' from 'A level and other 16 to 18 results'". <https://explore-education-statistics.service.gov.uk/data-tables/permalink/4000ba98-bbe6-4b65-b26c-508c06b578fb>

46. Parsons, R., and Britton, P. (2022) "'Shocking' North-South education divide revealed as teens in three Greater Manchester areas left with NO A-Level options". Manchester Evening News. <https://www.manchestereveningnews.co.uk/news/greater-manchester-news/shocking-north-south-education-divide-236504442>

An important factor to note is that the vast majority of mathematical sciences degrees currently require the very highest grades in A-Level Maths. Over 95% of those who study mathematical sciences degrees now go to institutions that require grade A or A* in A-Level Maths, and often A-Level Further Maths or other bespoke tests. 10 years ago, this proportion was closer to 85%.⁴⁷ The remaining institutions usually require grade B in A-Level Maths, which many would argue still demonstrates a good level of mathematical ability.

This means that even the student who manages to access A-Level Maths, and has the desire and ability to study a mathematical sciences degree, must be able to gain the very highest grades to guarantee that they will be able to enter such a degree, particularly if they wish to study locally.

The few places where grade B will qualify a student to enter a mathematical sciences degree may not be at locations that are viable for them to attend. Again, this is in strong contrast with other subjects such as computer science, where it is very possible to find a local university offering degree courses at a lower tariff.

Some of the remaining maths courses that require grade B in A-Level Maths are closing for lack of applicants, which is exacerbating the national problem. Yet the quality of these courses is often high, with evidence that their economic and employability outcomes are excellent. Given that A-Level grades are correlated with socioeconomic status, there is an access and participation issue here, if talented mathematicians are 'frozen out' of maths degrees by tariff barriers.

Overall, if we want more mathematical sciences graduates, which will, in turn, provide more qualified maths teachers, we need to provide opportunities for a greater proportion of the 16-18 cohort to take A-Level Maths. We must support students to achieve their potential through good teaching, and we need to find ways to encourage the takeup of mathematical sciences degrees across the whole tariff range.

47. UCAS. 2023 "Undergraduate sector-level end of cycle data resources 2022". <https://www.ucas.com/data-and-analysis/undergraduate-statistics-and-reports/ucas-undergraduate-end-cycle-data-resources-2022>



Russell Hobby

Chief Executive, Teach First

ADDRESSING THE TEACHER CHALLENGE

A plan for maths to 18 is also a plan for teachers. We need to take proper consideration of the workforce challenge in all schools, including primary schools, where teachers educate 4.6 million children in the foundations of mathematics. To give every child a specialist maths teacher, we would need to recruit 20% of all maths graduates in England. Even if we expand this to those with A-Level Maths, we would need 6% of all those who achieved grade A or A*. Significant efforts need to be put into place for primary and secondary teachers to support an expansion of maths to 18.

There is no viable strategy for maths excellence, at any age range, that does not factor in maths teaching and teachers. Although we learn about maths in many places, and some talented people will master it regardless, for most of us it is our maths lessons in school and college that lay the crucial foundations.

This is common sense, but it is often neglected in talk about our ambitions and aspirations. A maths excellence plan is, first and foremost, a teaching workforce plan. And the basic arithmetic here, if you forgive the glaring pun, is worrying.

- In 2022/23, there were 4.6 million students learning maths in primary school in England. We cannot neglect primary education.
- There were 3.6 million students learning maths between 11-16, including around 700,000 that take GCSE Maths every year.⁴⁸
- There are around 90,000 students that take A-Level Maths every year, and many more that take various other forms of maths, including GCSE resits, in further education.⁴⁹
- There are around 11,000 young people that go on to study an undergraduate maths degree at university.⁵⁰

As well as future architects, computer scientists, engineers, managers, small business owners and technicians, these students are also the source of future maths teachers. So the shortage in maths talent, in part driven by a shortage of maths teachers, leads on to precisely fewer of the teachers in the very area that we seek to improve.

Primary teachers teach maths alongside every other subject. It is worth pausing to reflect on just how much we expect primary teachers to be good at! The importance of primary teachers is not just that they teach children, it is also that they lay the foundations for maths excellence. And playing catch up for those who fall behind at primary level is much harder than getting it right first time.

In secondary schools, we need around 3,000 maths teachers to train, qualify and arrive in schools every year. Currently, it looks like the system will reach about 60% of this year's target. And this is where teacher shortages become a matter of social justice. When things are scarce, it is usually the schools serving low-income communities who miss out. This is certainly true when it comes to teacher recruitment, as the work is harder and the working conditions more challenging.

Teachers in the most disadvantaged secondary schools are more than twice as likely to believe that their departments are not adequately staffed as those in the most affluent schools (30% versus 14%).⁵¹ So, not only do teacher shortages prevent every child enjoying the mastery of mathematics, not only do they damage our economy and put a brake on growth, but they also drive a deeper wedge between the rich and poor of our country.

This injustice is the motive force behind Teach First, which aims to put the status of maths teaching on a par with other graduate professions. It is the highest-ranked purpose-driven recruiter in the Time Top 100 graduate employers. We aim to attract more of our best people into teaching and, above all, to ensure that they teach in schools serving low-income communities.

Teach First trained 165 maths teachers last year. Teach Firsters represent almost 10% of all new maths teachers and over 30% of those who started on postgraduate employment routes in schools serving low-income communities. They are also more likely to achieve qualified teacher status than trainees on any other major route.

But we should be in no doubt about the scale of the challenge facing us, and the school system generally. To give every child a specialist maths teacher, we would need to recruit 20% of all maths graduates in England. And we would need to do this in the face of starting salaries for mathematicians in the City and elsewhere, which can easily be double those for teachers, as well as opportunities for flexible working that are not possible for most teachers within schools. This is not going to happen under current conditions.

48. Ofqual. (2023) op. cit.

49. JCQ. (2022) "GCE A-level trends – Summer 2022, UK". <https://www.jcq.org.uk/wp-content/uploads/2022/08/GCE-Trends-Summer-2022-v2.pdf>

50. Higher Education Statistics Agency. (2023) "Higher Education Student Statistics: UK, 2021/22 - Subjects studied". <https://www.hesa.ac.uk/news/19-01-2023/sb265-higher-education-student-statistics/subjects#:~:text=First%20year%20enrolments%20in%20business,%2F21%20and%202021%2F22>

51. Allen, B., and McInerney, L. (2019) "The recruitment gap: Attracting teachers to schools serving disadvantaged communities". The Sutton Trust. <https://dera.ioe.ac.uk/id/eprint/33813/1/The-Recruitment-Gap.pdf>

It is, however, perfectly possible to teach maths up to GCSE with a good A-Level in the subject. Even so, we would need 6% of all those who achieved grade A or A* grade to enter teaching.⁵² This is more feasible in terms of numbers, but people with A-Level Maths often feel less confident about teaching it than those with degrees. To encourage more of those who have A-Level Maths to go into teaching, we should invest heavily in subject knowledge enhancement and ongoing continuing professional development, including subject specific Early Career Framework provision, which, although logistically challenging, is possible. The essay from Marie Hamer and Emma Lark speaks well to this point. As well as recruitment, we cannot neglect retention. Around 10% of teachers leave the profession every year.⁵³ Addressing this will reduce the recruitment targets and protect valuable expertise.

The good news is that the things we could do to improve maths teacher recruitment and retention would work for all subjects. Increase salaries. Reduce workload, especially less fulfilling tasks such as admin and marking. Improve the status of and respect for teachers. Smarten up how schools are held accountable. Support schools to develop flexible working policies. Support schools in managing student behaviour and constructive parental engagement. Consider the potential for AI to support teacher workload by, for example, reducing marking, preparing tests and exercises, and even supplementing teaching with personalised tuition. This latter solution could help students who have missed lessons or fallen behind, and who lack the foundational concepts or fluency that the rest of the class take for granted.

We also suggest additional support for shortage subjects like maths and for schools serving low-income communities. This could be an extension of the current bursary arrangements, targeted at retention and linked to working in schools serving high levels of Pupil Premium students. This makes the best use of limited resources. Financial incentives need to be relatively large to make a difference but do seem to work. Improving the level of funding for schools serving low-income communities would help them to create better working conditions and invest in professional development and career progression. A particularly valuable intervention is reducing the amount of contact time to increase capacity for planning and preparation. This also requires funding, and more teachers, so that it does not reduce student lesson time.

And, as mentioned earlier, we cannot neglect primary, where many of the same arguments for what we can do to boost teacher supply hold true. There is also an argument for a cohort of funded maths specialist teachers based permanently in any school that is struggling to reach expected standards in numeracy, to supplement the work of existing teachers. This would be a good, and perhaps more sustainable, enhancement of the National Tutoring Programme. I cannot stress enough that, however excited we are by visions of maths excellence among older age groups and in preparation for amazing career opportunities, it is a house of cards unless we help primary schools to get the basics right for all.

Maths is fascinating and at times beautiful. It crops up in everyday life, and in the farthest reaches of intellectual endeavour and technological progress. Competence for all and mastery for many, regardless of background and wealth, is a prerequisite to a just, thriving nation. And it all comes back to maths teachers. With a full complement of talented, trained and motivated maths teachers in every school, we can unpick the challenges of curriculum, pedagogy, assessment and catch-up with a high degree of confidence. Do not let any politician or policymaker talk to you about their plan for maths without also telling you about their plan for teachers.

52. JCQ. (2022) op. cit.

53. Gov.uk. (2023) "School Workforce in England, Reporting year 2022". <https://explore-education-statistics.service.gov.uk/find-statistics/school-workforce-in-england>



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TEACHER PROFESSIONAL LEARNING: AN EVIDENCE-LED APPROACH TO IMPROVING OUTCOMES

The success of maths to 18 hinges heavily on supporting existing maths teachers to be as effective as possible. All students need to have access to high-quality maths teaching throughout their schooling. Investing in professional learning for existing teachers of maths is a cost-effective and viable approach. Our 'best bet' for improving maths teaching is to incorporate the 'active ingredients' of effective professional learning, address gaps in mathematical knowledge and cover core teaching strategies. This might take the form of maths-specific teaching excellence programmes, with maths-specific exemplification throughout. One way to secure these improvements is through the continuation of recent education reforms in teacher development for all subjects.

At Ambition Institute, we wholeheartedly support the policy ambition of increasing access to high-quality maths education until age 18. Securing great maths outcomes is paramount for both individuals and society. Improved competence in maths has a range of benefits, including enhanced problem solving, stronger economic growth, increased equity in a range of ways and more sophisticated citizenship.⁵⁴

Turning this ambition into reality is agreed by supporters and critics alike to require a sufficient quantity of teachers, trained to be excellent teachers of maths, at all stages of our education system. Contrary to the received wisdom that such a scenario is implausible, we set out suggestions on how to lay the ground for such a policy to be successful.

First, the success of any such policy shift hinges heavily on supporting existing maths teachers to be as effective as possible. Numerous studies have highlighted the link between teaching quality and student attainment, with maths teaching and learning being a major contributor to this empirical base, due to its measurability and prevalence.⁵⁵

On average, teachers in the top third of effectiveness enable their students to learn at double the rate of those in the bottom third.⁵⁶ Effective teachers not only improve attainment, but also increase the likelihood that students attend school, progress to sixth form or college, earn more and live healthier, happier lives.⁵⁷ The most effective teaching also tends to benefit the least advantaged students the most.⁵⁸

This is even more important because the alternative, recruiting significantly more mathematics graduates, is currently an unfeasible option, as Russell Hobby sets out in his essay.⁵⁹ Annual maths teacher initial training targets have not been met for over a decade, despite targets being cut by 39% between 2019/20 and 2022/23.⁶⁰

Furthermore, attrition rates amongst maths teachers during their first five years are at their highest ever: 13% of maths teachers leave the state-funded sector after just five years, compared with 9.3% across all subjects.⁶¹

54. Hoyles, C., et al. (2010) "Improving mathematics at work: The need for techno-mathematical literacies". Routledge;

Hanushek, E. A., et al. (2015) "Returns to Skills Around the World: Evidence from PIAAC". *European Economic Review* 73 pp103-130. <https://www.sciencedirect.com/science/article/pii/S0014292114001433>;

Noyes, A., et al. (2023) op. cit.;

Gutstein, E. (2006) "Reading and writing the world with mathematics: Toward a pedagogy for social justice". Routledge.

55. Burgess, S. (2019) "Understanding teacher effectiveness to raise pupil attainment". *World of Labor*. <https://wol.iza.org/uploads/articles/515/pdfs/understanding-teacher-effectiveness-to-raise-pupil-attainment.pdf>

56. Wilian, D. (2016) "Leadership for Teacher Learning: Creating a Culture Where All Teachers Improve So That All Students Succeed". *Learning Sciences International*;

Hanushek, E. A. (2010) "The economic value of higher teacher quality". *National Bureau of Economic Research*. https://www.nber.org/system/files/working_papers/w16606/w16606.pdf

57. Heckman, J. J. (2006) "Skill formation and the economics of investing in disadvantaged children". *Science* 312 pp1900- 1902. https://jenni.uchicago.edu/papers/Heckman_Science_v312_2006.pdf;

Koedel, C. (2008) "Teacher quality and dropout outcomes in a large, urban school district". *Journal of Urban Economics* 64 (3) pp560-572. <https://www.sciencedirect.com/science/article/pii/S0094119008000594>;

Jackson, C. K. (2012) "Non-cognitive ability, test scores, and teacher quality: Evidence from 9th grade teachers in North Carolina". *National Bureau of Economic Research*. <https://www.nber.org/papers/w18624>

58. Chetty, R., et al. (2014) "Measuring the impacts of teachers II: teacher value-added and student outcomes in adulthood". *American Economic Review* 104 (9) pp2633-79. <https://www.aeaweb.org/articles?id=10.1257/aer.104.9.2633>;

Jackson, C. K., et al. (2020) "Who benefits from attending effective high schools?". *National Bureau of Economic Research*. <https://www.nber.org/papers/w28194>

59. Wilian, D. (2016) op. cit.

60. McLean, D., et al. (2023) "Teacher Labour Market in England Annual Report 2023". *National Foundation for Economic Research*. <https://www.nfer.ac.uk/teacher-labour-market-in-england-annual-report-2023/>

61. Worth, J. (2018) "Teacher Workforce Dynamics in England: Nurturing, supporting and valuing teachers". *National Foundation for Economic Research*. https://www.nfer.ac.uk/media/3111/teacher_workforce_dynamics_in_england_final_report.pdf

We propose that a more cost-effective and viable approach is to invest in professional learning for existing teachers of maths, including non-specialist teachers and those who lead maths departments.⁶² High-quality professional learning has been shown to be twice as effective as other school-level factors at enhancing student achievement. It has a similar effect on teacher quality as 10 years of classroom experience for an average teacher.⁶³

One US study found that elevating the weakest teachers to the level of an average teacher would have an estimated present value of \$100 trillion. Survey evidence suggests that teachers value one additional day of effective professional learning as much as a 0.4% increase in pay.⁶⁴ Given its appeal to teachers, enhancing professional learning could both increase the effectiveness of serving teachers, and likely mitigate challenges in teacher recruitment and retention.⁶⁵

Only certain forms of professional learning yield consistent improvements to teaching, and some can be detrimental.⁶⁶ Researchers have identified several 'active ingredients' of effective maths teacher development, including: securing participant buy-in; providing clear, evidence-informed conceptual models; providing examples of effective practice; giving teachers opportunities to practice applying these insights and strategies; providing concrete and actionable feedback; and ensuring that any changes to understanding and practices persist.⁶⁷ Incorporating these 'active ingredients' into professional learning will be vital to achieving maths to 18.

Effective professional learning for maths teachers must emphasise mathematical knowledge development. Only a third of those teaching maths to students aged 16-18 have a maths degree, and a third only have a Level 2 maths qualification (equivalent to a GCSE).⁶⁸ Only 44% of secondary maths teachers have a maths degree and teacher recruitment shortages have resulted in many non-specialists teaching mathematics.⁶⁹ Around 45% of secondary schools, and up to 62% of schools that are struggling to recruit, have used teachers without a maths degree for at least some maths lessons.⁷⁰

62. Hanushek, op cit.

63. 63 Allen, J. P., et al. (2015) "Enhancing secondary school instruction and student achievement". *Journal of Research on Educational Effectiveness* 8 (4) pp475-489. <https://www.tandfonline.com/doi/full/10.1080/19345747.2015.1017680>;

Opper, I. M. (2019) "Teachers matter: Understanding teachers' impact on student achievement". RAND Corporation. https://www.rand.org/pubs/research_reports/RR4312.html;

Fletcher-Wood, H., and Zuccollo, J. (2020) "Evidence review: The effects of high-quality professional development on teachers and students". Education Policy Institute. https://epi.org.uk/wp-content/uploads/2020/02/EPI-Wellcome_CPD-Review__2020.pdf;

Kraft, M., and Papay, J. (2014) "Can professional environments in schools promote teacher development? Explaining heterogeneity in returns to teaching experience". *Educational Effectiveness and Policy Analysis* 36 (4) pp476-500. <https://scholar.harvard.edu/mkraft/publications/can-professional-environments-schools-promote-teacher-development-explaining>

64. Hanushek, op cit; Burge, P., et al. (2021) "Understanding Teaching Retention: Using a discrete choice experiment to measure teacher retention in England". RAND Corporation. https://www.rand.org/pubs/research_reports/RR181-1.html

65. Hanushek, E. A. (2011) "The Economic Value of Higher Teacher Quality". *Economics of Education Review* 30 pp466- 479. <https://hanushek.stanford.edu/sites/default/files/publications/Hanushek%202011%20EER%2030%283%29.pdf>;

Chiong, C., et al. (2017) "Why do Long-Serving Teachers Stay in the Teaching Profession? Analysing the Motivations of Teachers with 10 or More Years' Experience in England". *British Educational Research Journal* 43 (6) pp1083-1110. <https://bera-journals.onlinelibrary.wiley.com/doi/full/10.1002/berj.3302>;

Casely-Hayford, J., et al. (2022) "What Makes Teachers Stay? A Cross-Sectional Exploration of the Individual and Contextual Factors Associated with Teacher Retention in Sweden". *Teaching and Teacher Education* 113 103664. <https://www.sciencedirect.com/science/article/pii/S0742051X2200035X>

66. Cordingley et al. (2015) "Developing great teaching: lessons from the international reviews into effective professional development". Teacher Development Trust. <https://dro.dur.ac.uk/15834/>;

Fletcher-Wood, H., and Zuccollo, J. (2020) op. cit.

67. Sims, S. et al. (2021) "What are the characteristics of effective teacher professional development? A systematic review & meta-analysis". Education Endowment Foundation. <https://d2tic4wvo1iusb.cloudfront.net/production/documents/pages/Teacher-professional-development.pdf?v=1689167897>

68. Noyes, A., et al. (2018) "A survey of teachers of mathematics in England's Further Education Colleges". Nuffield Foundation. <https://www.nottingham.ac.uk/research/groups/crme/documents/mifec/interim-report.pdf>

69. Department for Education. (2023) "School Workforce Census 2023". https://assets.publishing.service.gov.uk/government/uploads/system/uploads/attachment_data/file/1152210/2023_School_Workforce_Census_Specification.pdf

70. Worth, J., and Faulkner-Ellis, H. (2022) "Teacher supply and shortages". National Foundation for Economic Research. https://www.nfer.ac.uk/media/5143/teacher_supply_and_shortages.pdf

It is true, however, that higher maths qualifications for teachers have surprisingly little effect on student outcomes.⁷¹ Maths teacher effectiveness is not driven by their degree specialisation but how well equipped they are to teach maths. This relies on a combination of both strong mathematical knowledge and proficiency in core teaching skills, such as formative assessment.⁷²

In other words, professional development for existing maths teachers should not just mean more formal maths subject knowledge. Instead, our 'best bet' for delivering a robust improvement to maths teaching is a balanced approach that incorporates the 'active ingredients' of effective professional learning, addresses gaps in mathematical knowledge and covers core teaching strategies.

In practice, this might take the form of maths-specific teaching excellence programmes that focus on core teaching skills, with maths-specific exemplification throughout. For those without a maths degree, knowledge enhancement could focus on mathematical knowledge relevant to the higher grades at GCSE and to post-16 maths courses, and crucially also include training on pedagogical approaches to teaching maths at this level.

Finally, high-quality professional learning targeted exclusively at those teaching maths to students aged 16-18 is unlikely to yield strong and lasting improvements in maths outcomes. For maths to 18 to be effective, we need all students to have access to high-quality maths teaching throughout their schooling. This is because mathematics is a hierarchical subject, in which every stage of learning builds on the foundations that came before.

To make maths to 18 a reality for children, and to build a strong future for our country, it is imperative that any investment in professional learning is extended to maths teachers across all phases of education.

One of the best ways to secure such improvements to maths teaching is to ensure the continuation of recent education reforms in teacher development for all subjects, including those to Initial Teacher Training, the introduction of the Early Career Framework and the reform of National Professional Qualifications.

Together, these initiatives form a thread of professional learning throughout a teacher's career, covering core teaching and leadership approaches. Building on these, it would be beneficial to develop a progression pathway of evidence-led programmes targeted at all maths teachers, not only those at the start of their career or taking up leadership posts, which would be aimed at improving both teachers' mathematical knowledge, techniques and core teaching skills.

Investing in such evidence-informed professional learning for our teachers of maths is crucial to realising the policy ambition of extending high-quality maths education to 18. Such an investment represents a moral imperative, given its potential to transform student outcomes, enhance social mobility and strengthen our economy.

71. Wilian, D. (2016) op. cit.

72. Or similar constructs, such as Mathematical Knowledge for Teaching (MKT); Miller, D. I., et al. (2022) "Teachers' Pedagogical Content Knowledge in Mathematics and Science". American Institute for Research. <https://www.air.org/sites/default/files/2022-05/Teachers-Pedagogical-Content-Knowledge-in-Math-and-Science-April-2022.pdf>;

Hill, H. C., et al. (2008) "Mathematical Knowledge for Teaching and the Mathematical Quality of Instruction: An Exploratory Study". *Cognition and Instruction* 36 (4) pp430-511. <https://psycnet.apa.org/record/2008-15892-002>;

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Griffin, C. C., et al. (2009) "Novice special educators' instructional practices, communication patterns, and content knowledge for teaching mathematics". *Teacher Education and Special Education* 32 (4) pp319-326. <https://journals.sagepub.com/doi/10.1177/0888406409343540>;

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