Presumed Knowledge for Higher and Standard Level IB Diploma Mathematics at Birkerød Gymnasium

Number and Algebra

1. Order of algebraic operations. *Examples*: $2(3+4\times7) = 62$; $(-3)^2 = 9$; $-3^2 = -9$
2. Fluency in use of addition, subtraction, multiplication and division using integers, decimals and fractions (both with and without calculator).
3. The number systems: natural numbers (N); integers (Z); rationals (Q) and irrationals; real numbers (R).
4. Prime numbers and factors, including prime factorisation, greatest common factors and least common multiples.
5. Numbers in 'scientific notation': 1.22×10^5, 7.4×10^{-3}, including calculations involving them (both with and without calculator).
6. Rounding; decimal approximations and significant figures; percentage errors.
7. Absolute value (modulus): $|-5| = 5$ etc.
8. Intervals on the real number line using set notation and using inequalities. Example: $x \in [2;7[\Leftrightarrow 2 \leq x < 7$
9. Square roots and surds (without a calculator):
    a. Addition, subtraction, multiplication: $\sqrt{27} + \sqrt{75} = 3\sqrt{3} + 5\sqrt{3} = 8\sqrt{3}$; $\sqrt{3} \times \sqrt{5} = \sqrt{3\times5} = \sqrt{15}$; $(1+\sqrt{3})(2+\sqrt{3}) = 5 + 3\sqrt{3}$.
    b. Quotients: $\dfrac{8}{3+\sqrt{5}} = \dfrac{8\cdot(3-\sqrt{5})}{(3+\sqrt{5})(3-\sqrt{5})} = \dfrac{24-8\sqrt{5}}{4} = 6 - 2\sqrt{5}$.
10. Exponents / powers:
    a. Simple positive powers: $2^3 = 8$; $(-3)^3 = -27$; $(-2)^4 = 16$
    b. Power 0 and negative powers: $7^0 = 1$; $7^{-2} = 1/49$.
    c. Fractional powers: $121^{1/2} = \sqrt{121} = 11$; $27^{1/3} = \sqrt[3]{27} = 3$.
    d. Use index laws: $\left(\dfrac{2}{3}\right)^{-3} = \dfrac{27}{8}$, $16^{\frac{3}{2}} = 64$, $\left(5x^{-2}\right)^{-1} = \dfrac{x^2}{5}$, $x^{-3} \times x^2 = x^{-1} = \dfrac{1}{x}$
11. Percentages and exponential functions
    a. Percentage increase and decrease. *Example*: adding 7 % by multiplying by 1.07; subtracting 10 % by multiplying by 0.9.
    b. Calculating percentage change. *Example*: going from 123 to 150 is adding 22 % since $(150-123)/123 = 0.22$ or since $150/123 = 1.22$.
    c. Undoing percentage change. *Example:* a price has gone up by 25 %. It is now 1000 kr. What was the price before? Solution: $x\times1.25 = 1000$, so $x = 1000/1.25 = 800$ (i.e. 20 % less).
    d. Successive percentage change and compound interest. *Example:* I loaned $ 2000 from the bank. After 6 years of not paying it back, I now owe $4200. What is the yearly interest rate on the loan? *Solution:* Amount has gone up by the factor $4200/2000 = 2.1 = 210$ %. Each year it has therefore gone up by the factor $\sqrt[6]{2.1} = 1.13163$, so yearly interest rate is 13.2 % (to 3 significant figures)
    e. Exponential growth and decay (radioactive decay, population growth etc.)

Expressions and formulae

12. Basic manipulation of simple algebraic expressions involving factorisation and expansion. *Examples*: $ab+ac = a(b+c)$; $(a\pm b)^2 = a^2+b^2\pm2ab$; $a^2-b^2 =(a+b)(a-b)$; $3x^2+5x+2 = (3x+2)(x+1)$.
13. Addition and subtraction of algebraic fractions with denominators of the form $ax+b$.

    Example: $\dfrac{2x}{3x-1} + \dfrac{3x+1}{2x+4} = \dfrac{13x^2+8x-1}{(3x-1)(2x+4)}$.

14. Evaluation of formulae. *Example*: if $T = 2\pi\sqrt{L/g}$, what is the value of T for $L = 2$ and $g = 9.8$?

15. Rearrangement of formulae. *Example*: using the formula for T above, find a formula for L.

P.T.O. →

Equations and inequalities

16. Solution of equations and inequalities in one variable. *Example*: $\dfrac{3}{7} - \dfrac{2x}{5} = \dfrac{1}{2}(1 - x) \Rightarrow x = \dfrac{5}{7}$.

17. Solution of simultaneous equations in two variables. *Example*: $\left.\begin{array}{c} 2x + 5y = 23 \\ 3x - 4y = 0 \end{array}\right\} \Rightarrow x = 4, y = 3$

18. The properties of $<, \leq, >, \geq$.   *Examples*: $a>b, c>0 \Rightarrow ac>bc;$ $a>b, c<0 \Rightarrow ac<bc.$

19. Quadratics
    a. Factorise quadratic expressions.
       Examples: $6x^2 - 3x = 3x(2x - 1);\ 5x^2 - 3x - 2 = (5x + 2)(x - 1);\ 16x^2 - 9 = (4x + 3)(4x - 3).$
    b. Solve quadratic equations by
       i. factorisation: $x^2+3x-28 = 0 \Rightarrow (x+7)(x-4) = 0 \Rightarrow x = -7$ or $x = 4.$
       ii. completing the square: $x^2 + 2x - 2 = 0 \Rightarrow (x + 1)^2 = 3 \Rightarrow x = -1 \pm \sqrt{3}$
       iii. using the quadratic formula $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.
    c. Solve 'text problems' requiring the setting up of a quadratic equation and then solving the equation (rejecting any solution deemed impossible).

Geometry

20. Geometry of simple plane figures, e.g. angles and area of triangles, quadrilaterals, polygons, circles.
21. The Cartesian plane: ordered pairs (x, y), origin, axes; mid-point of a line segment and distance between two points in the Cartesian plane.
22. The linear function $y = mx + c$: its graph, gradient (m) and y-intercept (c). *Example*: Find the equation of the line that passes through the points (4, 3) and (–2, 1).
23. Right angled triangles:
    a. Pythagoras' theorem and its converse.
    b. Simple trigonometry- sin, cos, tan.
    c. Simple applications for solving triangles.
24. Non-right angled triangles: sine and cosine rules and area of a triangle.

Probability and Statistics

25. Use the addition rule of probability for *mutually exclusive* events; understand and use the term *expected frequency;* draw and use tree diagrams; determine the probability that two or more independent events will both occur. Use simple conditional probability when combining events. *Example*: The probability of picking two blue balls out of a bag containing 3 blue and 6 yellow one after the other, without replacement.
26. Descriptive statistics: collection of raw data, display of data in pictorial and diagrammatic forms (for example, pie charts, pictograms, bar graphs and line graphs).
27. Calculate simple statistics from discrete or grouped data, including mean, median, mode, range and interquartile range.