Capabilities of multispectral satellite data in the estimation of the water column parameters

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Hyperspectral data on coastal zone

- The radiative transfer equation
  - It links the Apparent Optical Properties (AOPs) to Inherent Optical Properties (IOPs)
  - Its resolution is implemented in Hydrolight\(^1\)

- Development of a semi-analytical model\(^2\)
  - Allows the development of efficient inversion algorithms which estimate the parameters of the IOPs, the water column depth and information on the seafloor

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Water column depth estimation

Estimation performed from hyperspectral data acquired at Lee Stocking Island, Bahamas

Multispectral data on the coastal zone

- Two classical methods to retrieve information on the water column and on the seafloor.
  - Estimation thanks to empirical methods\(^4,5\)
  - Estimation thanks to a physical based model\(^6\)

- The physical based model needs at least five parameters to be estimated
  - Three for the water constituents
  - At least one for the seafloor model
  - The water column depth


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Does multispectral data bring enough information to estimate all these parameters?
Outline

1. Description of the problem
   - The radiative transfer model
   - The estimability function

2. Tested configurations
   - Sensors
   - Environmental parameters
   - Mathematical definition

3. Results & Discussions
   - Results
   - Analysis of the minimal direction

4. Conclusions
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Elements included in the model

- The spectral optical sensor measures radiance, $L(\lambda)$
- The atmospheric correction convert radiance into reflectance

$$R_{rs}(\lambda) = \frac{L_u(\lambda)}{E_d(\lambda)}$$

- Elements that affect the reflectance above the surface
  - The air/water interface
  - The water column
  - The seafloor

source: [Mobley2004]
The radiative transfer model

The air/water interface

• The formulation is given by

\[
R^{\text{model}}_{rs}(\lambda) = \frac{\zeta \, r^{\text{model}}_{rs}(\lambda)}{1 - \Gamma \, r^{\text{model}}_{rs}(\lambda)}
\]

with \( R^{\text{model}}_{rs}(\lambda) \), the remote sensing reflectance above the surface, and \( r^{\text{model}}_{rs}(\lambda) \), the remote sensing reflectance under the surface.

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The radiative transfer model

In the water column

- The semi-analytical model of Lee is used

\[ r_{rs}^{model} = r_{rs}^{dp} \left( 1 - e^{-(K_d + K_u^C)z} \right) + \frac{\rho}{\pi} e^{-(K_d + K_u^B)z} \]

with
- \( z \), the water column depth
- \( r_{rs}^{dp} \), the deep water remote sensing reflectance
- \( K_*^d \), the diffuse attenuation coefficient
- \( \rho \), the bottom reflectance spectrum

- The \( r_{rs}^{dp} \), \( K_*^d \) are evaluated thanks to attenuation and the scattering in the water column
In the water column

• The attenuation and the scattering in the water column are modeled with $^5,^6,^7$
  
  - $C_{phy}$, concentration in phytoplankton
  - $a^*_cdom(\lambda_0)$, attenuation due to CDOM at $\lambda_0 = 443$nm
  - $C_{nap}$, concentration in non-algal particles

• The bottom reflectance spectrum is considered as a linear mixture of 2 “pure” endmembers

$$\rho = \alpha \rho_1 + (1 - \alpha) \rho_2$$

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To conclude about the model

- There are 5 unknown parameters in the model used in this study $C_{phy}, a^*_{cdom}(\lambda_0), C_{nap}, \alpha, z$

- The model is clearly non-linear
The estimability function

**Estimability**

- Let consider a physical model \( \eta_X(\theta) = \{ \eta_{X_1}(\theta), \ldots, \eta_{X_N}(\theta) \} \), with \( \theta \), the vector of parameters and \( X = \{ x_1, \ldots, x_N \} \) a sampling design.
  - The physical model is, in this study, the radiative transfer model.
    \[
    r_{rs}^{model} = r_{rs}^{dp} \left( 1 - e^{-(K_d+K_u^C)z} \right) + \frac{\rho}{\pi} e^{-(K_d+K_u^B)z}
    \]
  - The sampling design is associated to the available bands.

Can we associate \( \eta_X(\theta) \) to an unique \( \theta \), i.e. let \( \theta' \in \mathcal{V}(\theta) \), does \( \eta_X(\theta') = \eta_X(\theta) \) imply \( \theta' = \theta \)?
Estimability function - definition

• Let define the estimability function, $E_{\eta X, \theta}$ as below

\[ E_{\eta X, \theta}(\delta) = \min_{\theta' \in \Omega, \|\theta' - \theta\| = \delta} \|\eta X(\theta') - \eta X(\theta)\| \]

• Construction of the estimability function

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In the space of parameters

In the space of the model
The estimability function

**Estimability function - definition**

- Let define the estimability function, $E_{\eta X, \theta}$ as below\(^8\)

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The estimability function

Estimability function - properties

- Let \( \mathbf{y} \) be measurement of \( \eta_X(\theta) \), the estimability function can be linked to the location of the global minimum, \( \theta_{LS}(\mathbf{y}) \), where

\[
\theta_{LS}(\mathbf{y}) = \arg \min_{\theta' \in \Theta} \| \eta_X(\theta') - \mathbf{y} \|^2.
\]

- The estimability function allows to compare sampling design, \textit{i.e.} sensors, in order to retrieve parameters of a specific model
  - For us, the water column and seafloor parameters
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Sampling designs

• The sampling design is set according to the sensors
  • Four sampling designs are studied: one hyperspectral and three multispectral configurations
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![Quickbird sampling design](image)
Sampling designs

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Model parameters

• The estimability function is computed for a specific parametrization of the model

• Results for a moderately turbid water will be presented
  
  • $C_{phy} = 1 \text{mg/m}^3$
  • $a_{cdom}^*(\lambda_0) = 0.1 \text{m}^{-1}$
  • $C_{nap} = 1 \text{g/m}^3$

• Five water depths are considered
  
  • very shallow ($z = 50\text{cm}$)
  • shallow ($z = 1\text{m}$)
  • moderately shallow ($z = 5\text{m}$)
  • optically deep ($z = 15\text{m}$)

• One type of seafloor
  
  • $\alpha = 0.5$
Norm definition

• The norm in the model space is defined as:

\[ \| \eta_X(\theta') - \eta_X(\theta) \|^2 = \frac{1}{N} \sum_{i=1}^{N} (\eta_{xi}(\theta') - \eta_{xi}(\theta))^2 \]

• In the parameters space

\[ \| \theta' - \theta \|^2 = \sum_i \left( \frac{\theta'_i - \theta_i}{\theta_i} \right)^2, \]

with \( \theta = (\theta_1, \ldots, \theta_5) = (C_{phy}, a^{*}_{cdom}(\lambda_0), C_{nap}, z, \alpha) \)
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Description of the results

- The optimization algorithm gives an upper and a lower bound for the estimability function $E_{\eta_X, \theta}(\delta)$ at each $\delta$.
Description of the results

- For visualization purpose, the bounds of $E_{\eta X, \theta}(\delta)$ are linearly linked.
Results for a moderately turbid water

Very shallow water

\[ E_{\text{vis}}(\delta) \]

- Hyperspectral
- WorldView2
- Quickbird
- Pleiades

0.0 0.2 0.4 0.6 0.8 1.0

April, 16th 2019
Results for a moderately turbid water

Very Shallow water

Shallow water
Results for a moderately turbid water

Very shallow water

Shallow water

Moderately shallow water

Vecteur 2019, Rimouski, Québec
Results for a moderately turbid water

Very Shallow water

Shallow water

Moderately shallow water

Optically deep water
Analysis of the estimability function

• It appears that the parameters of the radiative transfer model is generally not estimable with multispectral data
  • These preliminary results argue for increasing the number of spectral bands.

Can we identify parameters that cannot be estimated?
Minimal direction

- Let us note $\theta^*(\delta)$ defined as below:

$$\theta^*(\delta) = \arg \min_{\theta' \in \Theta, \|\theta' - \theta\| = \delta} \|\eta_X(\theta') - \eta_X(\theta)\|$$

- $\theta^*(\delta)$ provides the parameters that make $\eta_X(\theta^*(\delta))$ the closest to $\eta_X(\theta)$

- The minimal direction $\theta - \theta^*(\delta)$ indicates one direction that gives the minimum distance $\|\eta_X(\theta') - \eta_X(\theta)\|$.

- It indicates which compounds in $\theta$, i.e. which parameters, imply the smallest modification of $\eta_X(\theta)$ under the constraints $\|\theta' - \theta\| = \delta$. 
Minimal direction for the very shallow water case

**Hyperspectral**

![Graph showing minimal direction for Hyperspectral data]

**WorldView2**

![Graph showing minimal direction for WorldView2 data]

**Quickbird**

![Graph showing minimal direction for Quickbird data]

**Pleiades**

![Graph showing minimal direction for Pleiades data]
Analysis of the minimal direction

Minimal direction for the very optically deep water case

Hyperspectral

WorldView2

Quickbird

Pleiades
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Conclusions

• A method to analyze the ability to estimate of the water column parameters with spectral data has been presented
  • Only the analytical expression of the radiative transfer model is needed
  • no probabilistic assumption is necessary

• This method allows to compare spectral sensors for coastal applications

• The estimation of the parameters of the water column parameters including the water column depth is a challenging problem
  • Few measurements compared to the number of parameters to estimate
Conclusions

• This study will allow us to analyze the ability to estimate the parameters of the radiative transfer model according to the environmental parameters
  • As defining the conditions to realize properly SDB

• The minimal direction will be studied to evaluate its ability to ensure that available ancillary data can help to estimate the water column parameters.
Thank you for your attention